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The Multiplicity of Factions: Multi-Dimensional Ideal Points for Interest Groups & Members of Congress

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Abstract

America's two-party system and related political institutions generally collapse conflict toward a single left-right dimension. While previous work underscored how such forces belie actual latent disagreement across multiple preference dimensions, more recent methodological improvements and new data allow for greater analysis of both the optimal number of dimensions as well as the ideal points of individual actors on each dimension. The researchers apply these methods to a dataset of legislators' roll-call votes and interest groups' publicly observable positions on bills. Doing so demonstrates that in addition to the classic left vs. right dimension, American national political conflict is optimally characterized by dimensions concerning agriculture, conservation, and development, and industry versus privacy. Characterizing these dimensions and the actors that exemplify them informs speculation about potential latent factions in American politics that might be "released" if American political institutions were reformed to better encourage multiparty-ism.

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1. Introduction

Many policy questions are multidimensional. While all policy issues to some extent involve trade-offs between directly opposed values—e.g., between government intervention in the economy versus free market capitalism—multidimensional issues by definition touch on multiple such trade-offs simultaneously. If all actors cared about all dimensions equally, then multidimensional issues would simply force trade-offs across relevant dimensions. However, because different actors tend to prioritize different dimensions, each actor can gain on the issues (or issue dimensions) they prioritize by compromising on issues they prioritize less.

In legislative settings, of course, this is what is commonly known as vote-trading or logrolling. But such dynamics are by no means restricted to legislators or policymaking. For example, to the extent that party competition focuses on one major dimension of conflict, the presence of additional dimensions complicates the maintenance of party coalitions, creating opportunities wherein copartisans may encounter consistent disagreement (while cross-partisans may experience no such across-issue tensions). Thus, multidimensionality provides one basis for both intraparty breakdown and bipartisan cooperation. Finally, to the extent that the dimensions relevant to a given conflict can be manipulated by combining proposals together (e.g., into an omnibus bill or candidate campaign), they will be relevant to different sets of organized interests—and thus enable Schattschneiderian conflict expansion and the construction of diverse advocacy coalitions. Thus, multidimensionality is central, and perhaps even necessary, for a rich understanding of many important and frequently-observed features of politics.

Despite the central theoretical importance of multidimensionality, efforts to explicitly account for it in models of legislative politics began relatively recently. Theoretical work, expanding on Krehbiel (1998), has examined how multidimensional spaces make political systems with more "veto players" less likely to generate policy changes (Tsebelis 2002), while also providing opportunities for members with extreme preferences on one dimension to make policy gains (Hitt, Volden, and Wiseman 2017). Empirically-focused efforts to account for multidimensionality have demonstrated its value relative to unidimensional measures such as DW-NOMINATE (Lewis et al. 2019). This work has demonstrated that accounting for multidimensionality allows one to identify within-party cleavages (Aldrich, Montgomery, and Sparks 2014) and across-issue preference heterogeneity (Crespin and Rohde 2010; Roberts, Smith, and Haptonstahl 2016).

Yet while multidimensionality may be an important feature of legislators' voting decisions, it is all the more essential for understanding the politics of party *factions* and organized *interests*. Indeed, despite the primacy of party polarization and partisan competition in American politics, neither party is a monolith. Both of the two major parties are comprised of coalitions of group interests that represent distinct, and sometimes conflicting, sets of preferences (Bawn et al. 2012). Party factions not only make each party quite distinct in organizational structure (Grossman and Hopkins 2016), but the competition among them creates opportunities for political entrepreneurs to exploit mismatches between the preferences of currently dominant factions and those in a party's electoral base—see, for example, the rapid rise of the Tea Party, its domination of the Republican Party, and its culmination in the nomination of Donald Trump (Blum 2020; Heaney and Rojas 2015).

Among organized interest groups, multidimensionality in preferences may allow organizations to form partnerships of mutual beneficial exchange, particularly across economic and social sectors. Such diverse lobbying coalitions have been shown to influence legislative bargaining (Phinney 2017) and agenda-setting (Lorenz 2020) as well as regulatory rulemaking (Dwidar 2022a, 2022b). More broadly, if party factions and organized interests represent the building blocks of governing coalitions, then any speculation about the effects of various electoral reforms on creating a true multiparty system in the United States would be informed by accounting for multidimensionality more explicitly. To date, however, attempts to empirically characterize interest group preferences have been limited to a single dimension Crosson, Furnas, and Lorenz (2020), defending or challenging individual status quos (Baumgartner et al. 2009), or segmenting interest group advocacy to issue domains (Hansen 1991; Grossmann 2013). Thus, while multidimensionality is widely assumed in interest group politics, it has not been well-characterized in empirical examinations of interest group preferences.

In this paper, we explicitly examine the relevant dimensions of multidimensional conflict in Congress by estimating multidimensional interest group and legislator ideal points on the same scale. To do so, we leverage an important methodological advancement by McAlister (2021), who derives theoretical implications of multidimensional roll-call voting and introduces a Bayesian nonparametric estimation procedure (BPIRT) to facilitate empirical examinations of multidimensionality in legislative politics. A key innovation of the BPIRT procedure is that it both identifies the optimum set of dimensions and estimates each voter's ideal point within each dimension. This innovation allows the multidimensionality of position-taking data is allowed to "speak for itself" rather than being either imposed ex ante and/or limited via difficult-to-interpret rules of thumb. In doing so, we not only uncover several key dimensions of conflict among the interest group population (beyond the dominant left-right dimension of modern politics), but the inclusion of interest group data in estimation uncovers difficult-to-identify dimensions of conflict among *legislators*—dimensions that are masked by roll-call data alone.

We proceed as follows. First, we more closely examine the specific innovations of the BPIRT method and their advantages for characterizing interest groups' revealed preferences across multiple dimensions. Second, we argue that augmenting the congressional roll-call matrix with publicly observable interest group bill positions (Lorenz, Furnas, and Crosson 2020) is sufficient to estimate valid multidimensional interest group ideal points using the BPIRT method. After introducing the resulting multidimensional interest group and legislator ideal points, mIGscores, we use them as the basis for several significant descriptive inferences. First, we uncover and characterize three dimensions in legislator and interest group position taking, beyond the classic left-right spectrum. More specifically, we find that dimensions broadly concerning *agriculture*, *development and preservation*, and *industry versus individual rights* provide significant explanatory power. Second, after describing and discussing these dimensions at greater length, we further explore the interest "factions" that persist across the totality of our multi-dimensional ideal points, via cluster analysis. Here, we find not only that several stable cluster exist in multiple dimensions, but that they also are not necessarily reflective of the two-party division that dominates American politics.

2. Legislators, Parties, Interest Groups, and Multidimensionality

Generally speaking, models of both roll call voting (Poole and Rosenthal 1985; Clinton, Jackman, and Rivers 2004) and position-taking on roll call votes (e.g., Treier 2010, Bertelli and Grose 2011, Crosson 2019) are spatial in nature. That is, political actors are depicted as facing two alternatives, "yea" and "nay," that present them with two different possible sets of outcomes. In choosing between the alternatives, actors simply select the alternative that generates an expected outcome as close as possible to their ideal policies.

Formally, ideal points are estimated in the roll call scaling context assuming that legislators vote in a way that maximizes utility under quadratic loss:

$$\begin{aligned} U_i(\vartheta_j) &= -\|\boldsymbol{\omega}_i - \vartheta_j\|^2 + \eta_{ij} \\ U_i(\boldsymbol{\varphi}_j) &= -\|\boldsymbol{\omega}_i - \boldsymbol{\varphi}_j\|^2 + \nu_{ij} \end{aligned} \tag{1}$$

where η_{ij} and ν_{ij} are stochastic elements of the utility functions, $\boldsymbol{\vartheta}_j$ the location of the proposed bill in a common K-dimensional policy space, and $\boldsymbol{\varphi}_j$ the location of the current status quo.

Let $\boldsymbol{Y}$ be a matrix of expressed opinions and $y_{i,j}$ be the choice that legislator i makes on bill j : $y_{ij} = 1$ if organization i supports bill j and $y_{ij} = 0$ if she opposes bill j. Under the previous utility model, the probability that legislator i supports bill j can represented as:

$$P(y_{ij} = 1) = F(\boldsymbol{\lambda}_j' \boldsymbol{\omega}_i - \alpha_j) \tag{2}$$

where $F(\cdot)$ is the CDF associated with the chosen error structure, the difficulty parameter $\alpha_j = \frac{\boldsymbol{\vartheta}_j'\boldsymbol{\vartheta}_j - \boldsymbol{\varphi}_j'\boldsymbol{\varphi}_j}{\sigma_j^2}$, and the discrimination parameter $\boldsymbol{\lambda}_j = \frac{2(\boldsymbol{\vartheta}_j - \boldsymbol{\varphi}_j)}{\sigma_j^2}$.

While different estimation strategies such as NOMINATE and IDEAL approach identification somewhat distinctly, each necessitate that the number of preference dimensions be specified, in order for the model to become empirically tractable. However, as McAlister (2021) underscores, the "appropriate" number of dimensions is determined in a somewhat subjective fashion. That is, estimation procedures typically measure how much additional variation is captured by the introduction of a new preference dimension—and then stop adding new dimensions once improvements to fit diminish. Using such an approach, the threshold beyond which improvements to fit are no longer "sufficient" ultimately rests upon the researcher's discretion. This is especially problematic given that previous research has underscored how highly consequential issue dimensions may appear only occasionally on roll call records (McAlister 2021). Such issues are compounded by the fact that party leaders may actively craft roll call records so as to maximize perceived differences between parties—and minimize intraparty differences (e.g., Cox and McCubbins 1993; 2005). If, for example, Democrats disagree on the appropriateness of particular climate-policy measures, party leadership may simply filter out legislation that exposes such cleavages. Such filtration may limit the number of relevant votes found in roll-call data, but it does not diminish the policy importance of the issue area—nor the political importance of the issue to the Democratic coalition.

In order to circumvent the need for arbitrary and subjective determinations regarding the addition of new preference dimensions, McAlister (2021) argues that estimation methodologies should exhibit two key features. First, dimensionality should be directly testable under distributional assumptions, and not subjective thresholds. Second, in order to capture consequential dimensions that are possibly subject to gatekeeping, he argues dimensionality ought to be tested for at the *vote* level, rather than on an entire set of roll call or position-taking data. McAlister develops a new approach to estimating n-dimensional ideal points, Beta Process Item Response Theory (BPIRT), that embodies these characteristics.

The latter challenge addressed by BPIRT—testing for dimensionality at the vote level—is both well-suited for and itself aided by applying the methodology to interest group behavior. As noted at the outset, interests should in theory serve as perhaps the primary source of multidimensionality in American politics. Indeed, their appellation as "special" interests underscores the issue-focused and even parochial outlook typically attributed to organized interests. Greenpeace is primarily interested in environmental politics, the American Medical Association is drawn to health policy, the National Education Association takes interest in education policy—and so on. In each case, it makes little sense why organized interests would exhibit unidimensional "ideology"—or cross-issue constraint (Converse 2006)—in no small part because of such groups' lack of interest in other policy areas. In fact, inasmuch as interest groups *do* exhibit preferences that are well-characterized by a single preference dimension, such a pattern may provide evidence that polarizing influences in American politics, such as partisan competition (Lee 2016), have begun to engulf interest groups and their activities.

More than providing an important and interesting context within which to examine multidimensionality in American politics, however, interest group activity itself helps to address some of the challenges that McAlister and others have underscored with respect to the scaling of roll call data. That is, while gatekeepers in the House and Senate can prevent roll call votes on bills that may fracture their respective caucuses, they cannot exercise such control over interest groups. In fact, among the most common moments when interest groups stake out positions on legislation is at the time of introduction (Lorenz, Furnas, and Crosson 2020). As a result, by using BPIRT to jointly scale interest group position-taking and roll call data, we stand to learn not simply about the multidimensionality of interest group preferences, but also more about the potential multidimensionality of legislator positions than revealed with roll call data alone.

3. Data and Data Preparation

3.1 Data

To examine multidimensionality in interest group preferences, we add interest group bill positions to the congressional roll-call voting matrix. Because interest groups are not required to disclose such positions under national-level transparency laws, interest group bill positions for federal legislation can be taken only from their publicly-observable statements. The non-partisan transparency organization MapLight records such statements as expressed in committee hearing testimony, press releases, news and trade publications, open letters, and other documentation. MapLight had recorded over 130,000 interest group bill positions on over 5000 bills introduced before the 109th to 114th Congresses (Lorenz, Furnas, and Crosson 2020). We use these as the foundation of our analyses here.

One feature of the MapLight data is that some 88 percent of the bills on which MapLight recorded positions died in committee. This is important for our purposes here, because partisan agenda-setting procedures (especially in the House) tend to select bills that more readily map onto a single dimension of ideological conflict (Aldrich, Montgomery, and Sparks 2014). By observing a set of bills that were not specifically chosen for their single-dimensionality, our data are better able to observe the multidimensionality of interest group and legislator preferences.

Leveraging the BPIRT procedure discussed above (and detailed further in the technical appendix), we estimate multidimensional ideal points for both legislators and organized interest groups. In applying BPIRT to interest groups, however, one must make a series of additional considerations. First, unlike members of Congress, organizations that take relatively few positions that load strongly onto a particular dimension. That is, while legislators are more or less required to cast a vote on legislation, interest groups—at least at the federal level (see Thieme, n.d. regarding requirements at the state level)—face no such requirement. As a result, many interests do not take enough positions that would allow them "escape" the priors (see appendix), particularly in the application of BPIRT. Because the priors on all dimensions are centered around 0, an organization could exhibit an ideal point near the center on a given dimension either because they are truly centrist on that dimension, or because they did not take enough positions to impart requisite information for estimation.

Second, given that BPIRT is designed to detect more than one dimension, it necessarily requires items that impart more information than might be necessary for identifying a single dimension. That is, while all estimation techniques drop bills that are unanimous and overwhelmingly lopsided, BPIRT is potentially more sensitive to lopsided votes, requiring more information about the divided between yeas and nays.

In response to these challenges, we have parsed our interest group position-taking more stringently than Crosson, Furnas, and Lorenz (2020), imposing the following criteria. First, we require that legislators and interest groups take at least 15 positions—and that at least some of those positions are either yeas or nays (and not exclusively one or the other). Second, we require that legislators and interest groups take positions on bills in both the House and Senate, and not exclusively one chamber or the other. This decision derives from McAlister's finding that BPIRT picks up on differences in the House and Senate position-taking environments in a way that NOMINATE and IDEAL do not. Finally, remove all *bills* that themselves lack 15 positions total, and which lack sufficient balance between yea and nay positions. Doing so ensures that dimensions relying on smaller amounts of items can actually be identified.

After parsing our data according to these restrictions, we are able to generate mIGscores for 820 interest groups and 946 legislators, using 1,017 unique pieces of legislation. Importantly, the inclusion of interest group data enables us to utilize some legislation that never receives a roll call vote, similar to Crosson (2019).

4. Results & Discussion

In line with our expectations, the introduction of position-taking by "special" interests into congressional preference estimation ultimately yields more than a single dimension. In fact, our BPIRT scaling of interest group and legislator position-taking reveals five significant preference dimensions, the resulting scores for which we call multidimensional IGscores (mIGscores).

Below, we depict and discuss each of these dimensions, drawing out specific examples of groups and issues that illustrate the character of each dimension. Beyond these depictions, however, we also analyze how groups' revealed preferences cluster together *across* dimensions. As we demonstrate, latent groups of interests do appear to take positions in tandem across the revealed preference dimensions. We discuss these clusters at some length, underscoring how our present two-party system "flattens" some of these latent clusters.

Traditional Left-Right Dimensions: Dimensions 1 and 3

Before discussing the "new" dimensions uncovered by the application of BPIRT to joint interest group-legislator data, we first establish the "primary" dimension—and a related, though distinct second dimension—that our scaling recovers. That is, reassuringly, BPIRT recovers a first dimension (D1) that corresponds closely with the standard left-right dimension typically recovered by existing measures (e.g., NOMINATE). As previous scholarship has described, this dimension corresponds roughly with an "interventionist" versus "free-market" or "big" versus "small" government cleavage among elected officials (Poole and Rosenthal 1991). As Figure 1 depicts, this primary dimension correlates strongly with the single dimension IGscore estimated by Crosson, Furnas, and Lorenz (2020) using traditional Bayesian IRT—a dimension that itself corresponds quite strongly to that recovered by roll-call-only measures like NOMINATE for legislators. Here, House members' D1 mIGscores correlate with their original one-dimensional IGscores at 0.95, while interest groups' D1 mIGscores correlate with their IGscores at 0.93.[1]

Beyond this primary left-right dimension, however, a separate left-right dimension also appears as significant in our scaling: namely, a dimension that reveals key ideological differences in the left-right mappings of Senators and members of the House. Although a full discussion is beyond the scope of this paper, one feature of BPIRT is that it is able to detect potential behavioral differences between actors who are differently situated, in terms of their exposure to specific types of roll calls. Perhaps due to the more open amending environment in the Senate, our BPIRT scale recovers a

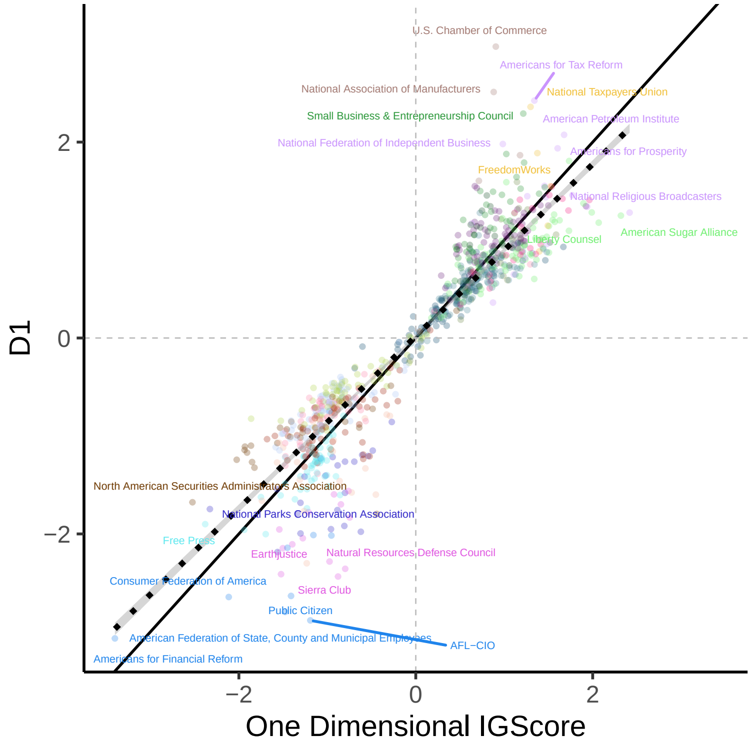

Figure 1. One-Dimensional IGscores against Dimension 1 (House L-R) of mIGscores

separate dimension—D3—that is statistically distinct from the primary left-right dimension, D1. In this dimension, Senators D3 preferences are correlated with original IGscores at 0.90.

D3 is plotted against unidimensional IGscores in Figure 2, Panel A. As the figure depicts, this Senate-specific dimension is quite distinct from unidimensional IGscores, particularly compared to D1. In fact, as Panel B of Figure 2 illustrates, the D1 and D3 are not especially well-correlated among interest groups. This suggests that Senate votes—both bills and amendments—and the processes underlying the consideration may reveal important, and distinct, information about political actors. Given that a great deal of scholarship on legislative politics presupposes similarities between legislators' preferences when serving in both chambers, we believe this finding constitutes an important area for further investigation.

Agriculture: Dimension 2

Beyond the standard left-right dimensions underlying much of American politics, our BPIRT scaling reveals a number of key secondary dimensions. Among the most influential of these dimensions deals with agriculture. Throughout much of American history, agriculture has made for interesting—and sometime strange—political alliances. First and foremost, given natural climate, soil, and human capital features necessary to render agriculture possible, commodity production itself carries with it a sort of regional factionalism. While southern states, for example, enjoy a climate amenable to tobacco,

1. Although we discuss our across-dimension cluster analysis at great length below, we note here that this figure and those that follow depict across-dimension clusters by color.

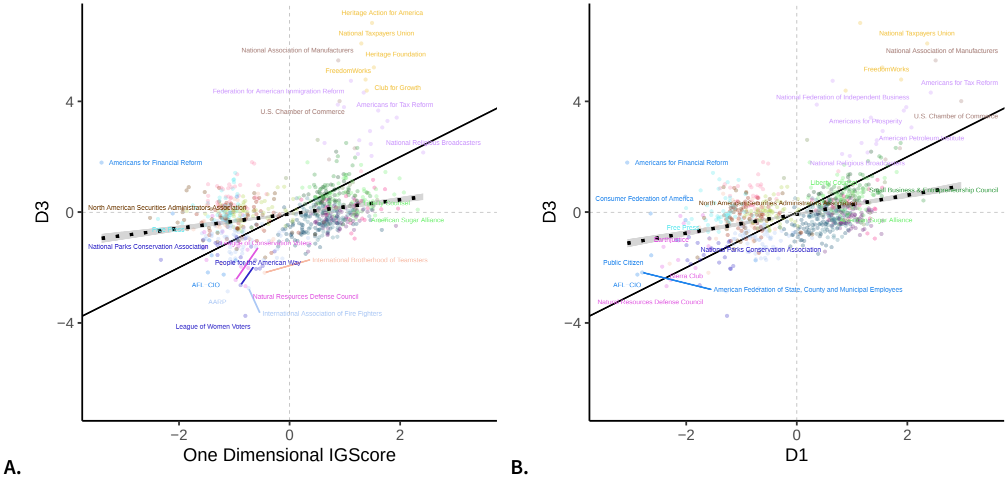

Figure 2. One-Dimensional IGscores against Dimension 3 (Senate L-R) of mIGscores

sugar beet, and cotton production, northern climates have long held a comparative advantage in dairy. Meanwhile, the rich, deep topsoil and open spaces of the Midwest and Great Plains lend those states well to the production of multi-use cash crops like soybeans—and especially crops that are tough on soil (yet fetch high prices) like corn. These geographical differences, in turn, lead to key economic differences, which have driven political wedges between these regions on issues like energy and foreign trade.

Of course, the oddities of agricultural politics extend far beyond differences in geography and commodity type. Indeed, particularly compared to political opinion in the rural areas within which farms are located, agricultural interests are decidedly moderate—or even left-leaning—on issues such as immigration. Long harvest seasons often necessitate at least seasonal help from migrant workers, with many farms drawing on immigrant labor for even longer periods of time. Consequently, although the racial, geographic, and socioeconomic features of farmers point toward strong allegiances with right-leaning parties, the unique economic situation of farmers creates interesting coalitions on immigration and related issues. It is perhaps unsurprising that farming interests have thus spent time in both the Republican and Democratic parties over time—and were even among the strong proponents of anti-party reforms during the Progressive Era.

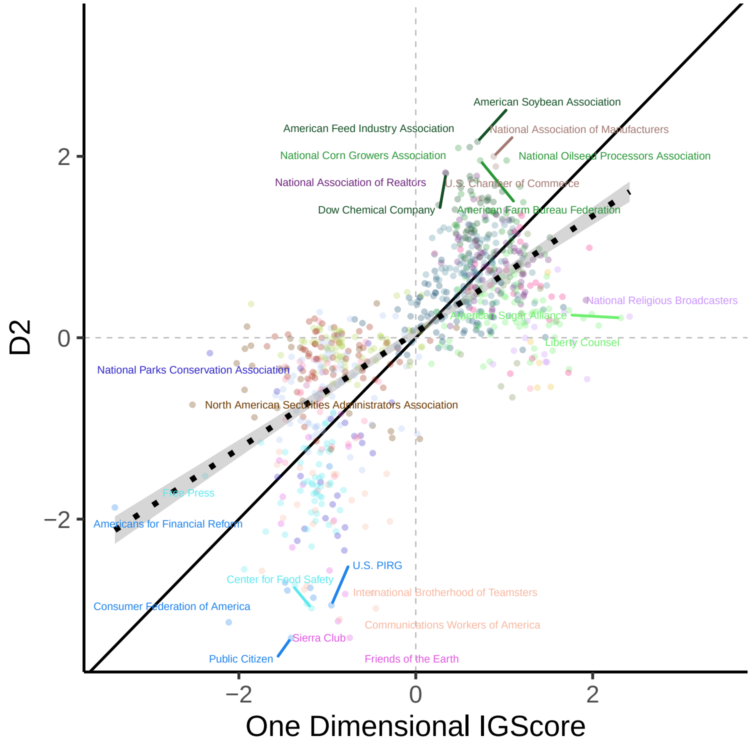

Figure 3. One-Dimensional IGscores against Dimension 2 (Agriculture) of mIGscores

At present, the historical oddities of agriculture politics have manifest as a generally bipartisan coalition on agricultural issues, which has grown out of Congress's pairing of food stamp and school nutrition programs with farm subsidies in the farm bill. With time, however, our findings appear to indicate that agriculture remains a cross-cutting issue below the surface. Indeed, as Figure 3 indicates, D2 is clearly "rotated" relative to one-dimensional IGscores. Moreover, there is a substantial difference in alignment between interest groups and members of Congress between D1 and D2. While the correlation between D1 and D2 is -0.78 among members of the House, the same correlation is *positive* 0.8 among interest groups.[2] Finally, while farming interests like the American Farm Bureau Federation registers a fairly moderate in a single dimension, for example, their position-taking is decidedly more one-sided when it comes to agricultural policy. Certainly, agriculture no longer employs the number of individuals it once did, particularly in comparison to the Progressive Era. However, the politics of agriculture, we argue, complicate modern-day political allegiances more than a single ideological dimension depicts.

Conservation vs. Development: Dimension 4

Similar to agriculture politics, environmental politics are notorious for introducing cleavages in party systems. Often represented by green parties in European political systems, the U.S. has lacked a

2. Any of the dimensions can be flipped, so the fact that it is specifically legislators' ideal points in D2 that are negatively correlated with their ideal points in D1 is arbitrary; the key observation is that actors' ideal points relative to one another are quite different across different dimensions.

singular green party throughout its history. However, with the rise of the conservationist movement, both Republicans and Democrats have played integral roles in increasing the government's role in environmental protection.

Although not *precisely* a true "environmental dimension", our mIGscores reveal a significant cleavage that we have termed the "Conservation vs. Development" dimension. This dimension, D4 in Figure 4, is strikingly distinct from preferences captured by unidimensional IGscores—nearly orthogonal in nature to IGscores' single dimension.

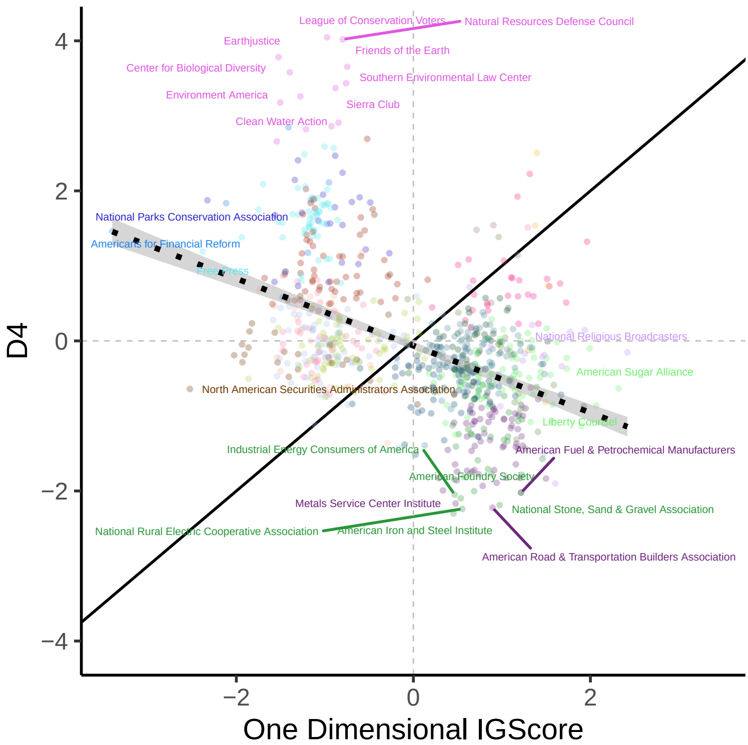

Figure 4. One-Dimensional IGscores against Dimension 4 (Conservation vs. Development) of mIGscores

Among the bills that load most heavily in this dimension, typical legislation focuses on trade-offs between development and economic growth versus environmental protection and conservation. Given this subject matter, there are several groups on each end of D4 that one might expect to observe. For instance, the American Iron and Steel Institute and the American Fuel & Petrochemical Manufacturers lie on the "development" portion of D4: not only are these industries' refining processes subject to considerable environmental regulation, but petroleum and steel both benefit from construction and business expansion into natural areas. Conversely, the "conservation" end features groups such as the League of Conservation Voters and the Environmental Defense Council. However, both ends of D4 also feature less obvious groups. The National Stone, Sand & Gravel Association, for example, is among the most "development" groups in D4, clearly benefiting from land development and road construction, rather than habitat preservation. FreedomWorks, by contrast, actually finds itself on the "conservation" end of D4, perhaps due to its libertarian leanings on issues like eminent domain.

Taken together, our findings clearly indicate that environmental-adjacent issues do introduce some cross-cutting cleavages into American politics. Certainly, those cleavages do not reach the levels seen in other industrialized nations. However, they may structure congressional policymaking in potentially underappreciated ways.

Industry vs. Individualism: Dimension 5

The final dimension, D5, is perhaps the most difficult to define substantively. This dimension, depicted in Figure 5, appears to correspond most strongly with business, industry, and globalization and the challenges of individual rights that it presents. Here, groups such as the U.S. Chamber of Commerce, the National Association of Manufacturers, and other corporate interests with connections across the world occupy the "industry" portion of D5. By contrast, the American Civil Liberties Union and Americans for Financial Reform occupy the "individualism" end of D5. Here again, though, both ends of the D5 dimension include potentially surprising organizations, underscoring key differences between D5 and typical left-right characterizations of American politics. Among the closest organizations to the ACLU, for instance, are both FreedomWorks and The Heritage Foundation, the latter of which is hardly considered "libertarian" in its leanings. On the other end of D5, the National Association of Manufacturers finds itself in the same neighborhood as the U.S. Conference of Mayors. One common way this distinction manifests in contemporary policy-making is in cybersecurity, information sharing, intellectual property, and privacy legislation. For example, the Stop Online Piracy Act, the Cybersecurity Information Sharing Act of 2015, and the Protecting Cyber Networks Act all exhibit high discrimination in position-taking along this fifth dimension.

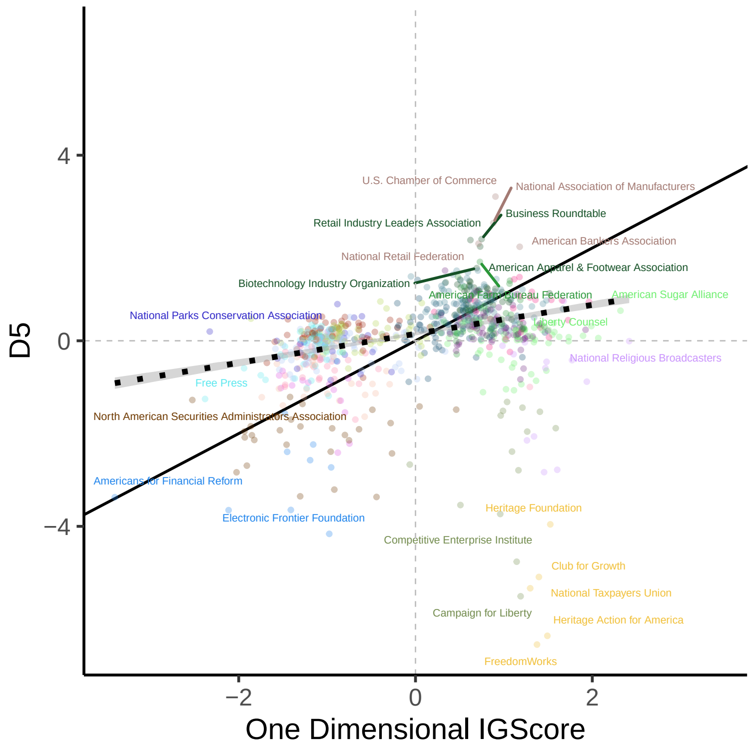

Figure 5. One-Dimensional IGscores against Dimension 5 (Industry vs. Individualism) of mIGscores

These oddities again highlight how our application of BPIRT to join interest group-legislator data uncovers both interesting and consequential cleavages underlying congressional policymaking. While parties may certainly prefer that interest groups situate themselves within one party coalition or the other, the diversity of political interests in the modern economy and 21st Century society can render such coalitions messy. To conclude, then, we examine more closely the clusters highlighted by the color differences in the preceding figures. In doing so, we attempt to make sense of which groups most consistently take similar positions across our 5 identified dimensions.

Latent Factions in American Politics

Table 1. 21 Interest Group Clusters

Cluster 1	**Cluster 8**	**Cluster 15**
Pesticide Action Network	Association of Global Automakers	Brennan Center for Justice
Women's Voices for the Earth	CA Technologies	Human Rights Watch
Alliance for a Just Society	Medtronic	Council on American-Islamic Relations
NETWORK, a National Catholic Social Justice Lobby	Applied Materials	Open Technology Institute
Cluster 2	**Cluster 9**	**Cluster 16**
Concerned Women for America	National Taxpayers Union	National Confectioners Association
American Family Association	Heritage Foundation	United Technologies
National Religious Broadcasters	FreedomWorks	Abbott Laboratories
John Birch Society	Club for Growth	National Council of Farmer Cooperatives
Cluster 3	**Cluster 10**	**Cluster 17**
Pew Charitable Trusts	National Cotton Council	Earthjustice
American Society for the Prevention of Cruelty to Animals	National Association of Wholesaler-Distributors	Greenpeace
OWL - The Voice of Midlife and Older Women	North American Equipment Dealers Association	Environment America
First Focus Campaign for Children	Portland Cement Association	Southern Environmental Law Center
Cluster 4	**Cluster 11**	**Cluster 18**
American Rental Association	United Steelworkers	Aircraft Owners and Pilots Association
Construction Industry Round Table	Consumers Union	National Association of Police Organizations
National Marine Manufacturers Association	Consumer Federation of America	National Association of Independent Colleges and Universities
International Sign Association	U.S. PIRG	American College of Emergency Physicians
Cluster 5	**Cluster 12**	**Cluster 19**
Heartland Institute	National Advocacy Center of the Sisters of the Good Shepherd	National Foreign Trade Council
Americans for Limited Government	Every Child Matters	Citigroup
R Street Institute	Democracy 21	National Chicken Council
Taxpayers Protection Alliance	Campaign Legal Center	Distilled Spirits Council of the United States
Cluster 6	**Cluster 13**	**Cluster 20**
CounterCorp	Southern Poverty Law Center	Textile Care Allied Trades Association
Union Plus	Center for Progressive Reform	American Staffing Association
U.S. Chamber Watch	Clean Air Task Force	Latino Coalition
Worksafe	Institute for Science and Human Values	National Automobile Dealers Association
Cluster 7	**Cluster 14**	**Cluster 21**
Arab American Institute	U.S. Chamber of Commerce	National Council of La Raza
USAction	American Bankers Association	Consortium for Citizens with Disabilities
National Priorities Project	National Retail Federation	BlueGreen Alliance
Children's Defense Fund	National Association of Manufacturers	American Sustainable Business Council

Our ultimate goal in this paper is to uncover latent interest group factions whose distinctiveness is elided under the current mode of two-party competition. This requires us to group together organizations that are similar to one another across the five dimensions we uncover with BPIRT. For this purpose, we employ K-Means clustering to mIGscores to identify 21 clusters.[3] The four organizations closest to each cluster's centroid—and thus most representative of its members—are

3. Like many other semi-supervised clustering algorithms, K-Means clustering requires the analyst to set the number of clusters to identify ex ante. We settled on 21 clusters here because that is the point at which additional clusters provided trivial additional goodness of fit, following Hartigan's Rule for additional cluster selection (Hartigan 1975).

listed in Table 1. These lists exhibit substantial face validity in the organizations they group together. For example, in cluster 14 are the U.S. Chamber of Commerce, the American Bankers Association, the National Retail Federation, and the National Association of Manufactures; in essence, cluster 14 represents peak business organizations (in fact, it is comprised *only* of these organizations).

The measure also distinguishes between clusters of organizations that share partisan or ideological lean but have distinct, and sometimes conflicting, identities. For example, contrast the peak-business interests of cluster 14 with the anti-redistribution groups making up cluster 9—including the National Taxpayers Union, the Heritage Foundation, FreedomWorks, and Club for Growth—and the more small-federal-government-oriented organizations in cluster 5—e.g., the Heartland Institute, Americans for Limited Government, the R Street Institute, and the Taxpayers Protection Alliance. Similar distinction happens among more generally left-leaning groups, with the environmental organizations in cluster 17—Earthjustice, Greenpeace, Environmental America, and the Southern Environmental Law Center—distinguished from rights-oriented organizations in cluster 15—e.g., the Brennan Center for Justice, Human Rights Watch, Council on American-Islamic Relations, and the Open Technology Institute—and the more specifically reform-focused organizations in cluster 13—including the Southern Poverty Law Center, the Center for Progressive Reform, the Clean Air Task Force, and the Institute for Science and Human Values. Distinctions between the more industry-specific clusters are similarly informative.

To demonstrate the face validity of these cluster estimates beyond the "top four" in each, we present a t-distributed stochastic neighbor embedding (t-SNE, see Van der Maaten and Hinton 2008) visualization of the organizations, colored by cluster, in Figure 6. The t-SNE takes higher-dimensional data (i.e., mIGscores) and reduces it to a lower-dimensional plot (Figure 6) that attempts to preserve the relative distance of each point from each other point, while nonlinearly balancing local and global structure.[4]. Figure 6 further validates the results of our K-Means clustering because organizations placed in the same cluster (as indicated by each point's color in Figure 6) are, with few exceptions, positioned relatively close to each other in the figure. In effect, this represents a form of convergent validity that leads us to be more confident that the clusters do in fact represent distinct subgroups of organizations.

Members of a given cluster are, by construction, more likely than other organizations to take positions on similar bills and to take similar positions on those bills. Given this basic point and the validity checks discussed above, we treat the 21 clusters as representing semi-coherent sets of organized interests—or, as we term them for efficiency here, factions. Treating each faction as a unit allows us to examine ideological alignment between factions as well as the kinds of resources the members of each faction bring to bear in their advocacy efforts. Table 2 shows the number of organizations in each cluster as well as its members' average IGscore, mIGscores (D1-D5), activity level, and PAC and lobbying expenditures (for the latter three, using data from Crosson, Furnas, and Lorenz 2020). This gives us a kind of "faction-level view" of conflict among organized interests.

Several observations immediately arise. First, as further demonstrated in Figure 7, most factions exhibit substantial heterogeneity in their ideal points (relative to other factions) across each dimension of the mIGscore. Perhaps in no pair of factions is this heterogeneity more starkly evinced than between the Peak Business (cluster 14) and Anti-Redistribution (cluster 9) groups. On both of the standard L-R dimensions(D1 & D3), these are the two most conservative factions. While Peak Business remains at roughly the same relative position across the dimensions, Anti-Redistribution swings wildly, simultaneously the most conservative faction on the Senate L-R dimension (D3) but siding with a mix of civil rights and libertarian groups and *opposite* Peak Business on the Industry vs. Individual (D5) dimension. The Labor-ish and anti-corporate faction in cluster 6—e.g., CounterCorp,

4. For a readable primer on interpreting t-SNE plot see Wattenberg, Viégas, and Johnson (2016)

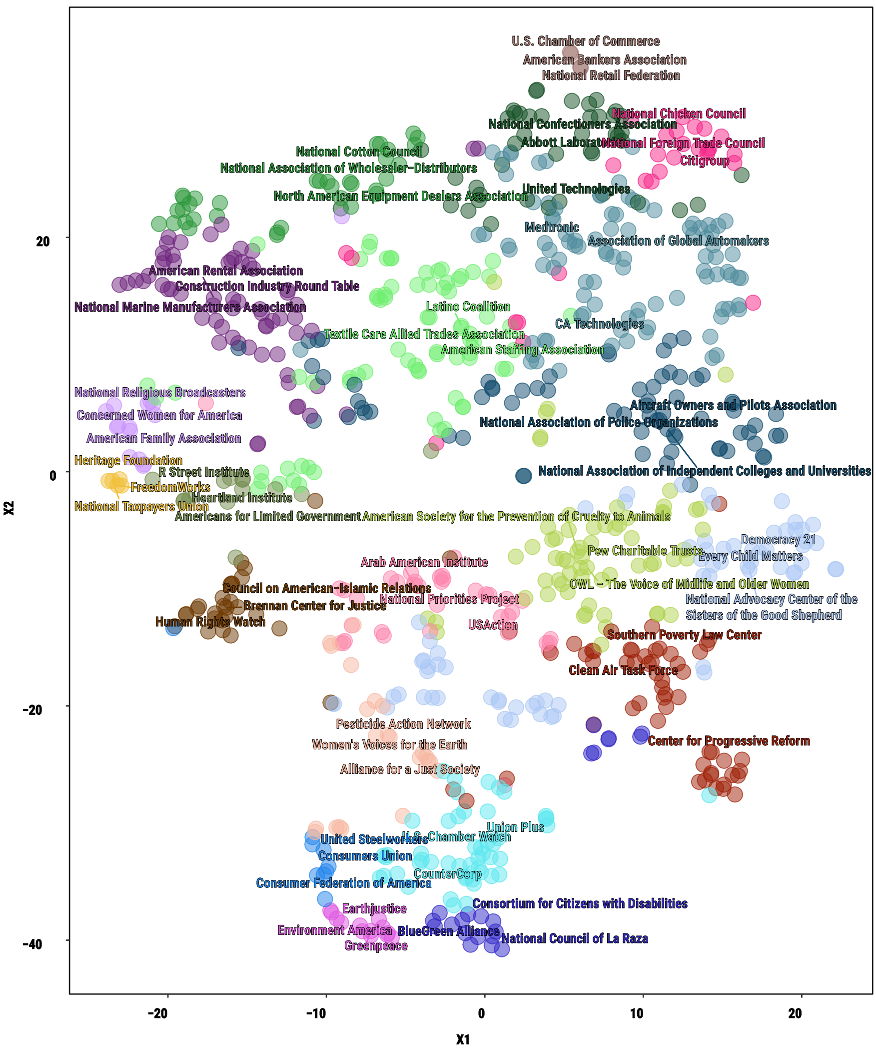

Figure 6. t-SNE of interest groups, colored by K-Means cluster

Union Plus, U.S. Chamber Watch, and Worksafe—similarly finds itself aligned with very different factions across different dimensions. This heterogeneity in relative positions implies a potential for different factional alignments—that is, coalitions—to emerge across different dimensions, and suggests that traditional ideological allies can be split depending on which dimension is most salient on a given policy question.

Second, the factions also vary substantially in their size, levels of activity, and expenditures on campaign contributions and lobbying. While the elite few Peak Business organizations dwarf everyone else on lobbying expenditures, the Anti-Redistribution groups spend considerable amounts of (presumably otherwise unredistributed) money on campaign contributions–again, dwarfing all other factions despite being relatively small in number. Indeed, many—but not all—factions appear to spend more on lobbying than PAC contributions as a mode of influence, suggesting that different types of interests might prefer different "modes" of advocacy for strategic reasons. This also suggests that there may be substantial heterogeneity on other relevant factional attributes. These might

Table 2. Summary of Interest Group Clusters

Cluster	N	IGscore	D1	D2	D3	D4	D5	Positions	PAC $	Lobby $
1	23	-1.00	-1.18	-2.01	-0.51	0.02	-0.82	87.60	939.98	331.40
2	15	1.42	1.39	0.16	3.13	-0.34	-0.75	117.17	839.28	1327.28
3	64	-0.73	-0.49	-0.08	-0.03	-0.12	0.11	51.50	12.08	267.41
4	64	0.90	1.01	0.86	-0.17	-1.33	0.30	56.93	202.52	1225.33
5	14	1.03	0.90	0.24	0.64	-0.35	-2.62	84.77	33.25	342.15
6	50	-1.19	-1.33	-1.70	0.07	1.63	-0.17	75.19	32.62	169.87
7	36	-0.98	-0.76	-0.63	0.66	0.05	-0.44	43.21	228.96	138.98
8	96	0.68	0.56	0.76	-0.14	-0.23	0.80	48.56	199.42	3927.70
9	5	1.42	1.57	-0.14	5.47	0.77	-5.46	221.40	1818.40	80.61
10	35	0.85	1.20	1.23	1.03	-1.41	0.56	98.22	208.64	1936.90
11	9	-1.57	-2.33	-2.64	-0.75	1.60	-2.93	299.57	766.06	720.59
12	70	-0.92	-0.68	-0.51	-0.96	-0.10	-0.14	52.62	107.59	838.78
13	55	-0.93	-0.88	-0.22	-0.16	1.01	0.16	50.08	132.65	169.27
14	4	0.92	2.24	1.70	3.43	1.17	2.44	603.00	1450.13	34526.80
15	28	-1.18	-0.91	-0.44	-0.27	-0.07	-2.06	78.00	0.10	261.15
16	44	0.72	0.76	1.35	0.52	-0.12	1.03	65.14	123.00	3492.04
17	13	-1.11	-2.02	-2.00	-1.29	3.35	-1.18	167.31	508.93	253.17
18	71	0.37	0.32	0.27	-0.58	-0.55	0.29	45.71	155.36	2022.92
19	28	1.07	0.91	0.59	0.13	0.75	0.61	47.23	154.66	2474.70
20	74	1.04	0.84	0.27	0.47	-0.48	0.08	39.51	273.80	959.63
21	22	-1.00	-1.53	-1.34	-1.59	1.61	-0.10	161.47	281.00	474.63

Notes: All columns except Cluster and N are cluster averages. Positions refers to the number of positions in the Crosson, Furnas, and Lorenz (2020) dataset. PAC spending and Lobby spending are scaled to increments of $10k. Position numbers, PAC, and Lobby data are currently missing for 79 organizations with mIGscores; this table will be updated in a future draft.

include the diversity of industries and other causes found in a faction, the coordination of a faction's members (e.g., the extent to which members take positions on the same bills), the cohesion of a faction's members (e.g., tendency to not be "split" on a given bill on which at least one takes a position, though this will necessarily be truncated by the clustering process), as well as judicial or regulatory lobbying or indicators of "outside" lobbying such as media and social media activity, membership totals, protest activity, and engagement with the judicial process. A key value-added of this clustering of factions will be in its ability to show how these differences that often emerge at the organizational level translate into the broader cleavages that define political conflict at the American national level.

5. Conclusion

What cleavages shape congressional politics and interest group conflict in the United States? Previous attempts at examining dimensionality in American national political conflict have shown that contemporary interest groups exhibit polarization across the same classic Left-Right dimension as members of Congress, but that apparent unidimensionality in Congress may be in part an artifact of partisan agenda-setting. Using new methods that both uncover latent dimensions in public positions in a principled way and estimate group ideal points across each of those dimensions, we have uncovered

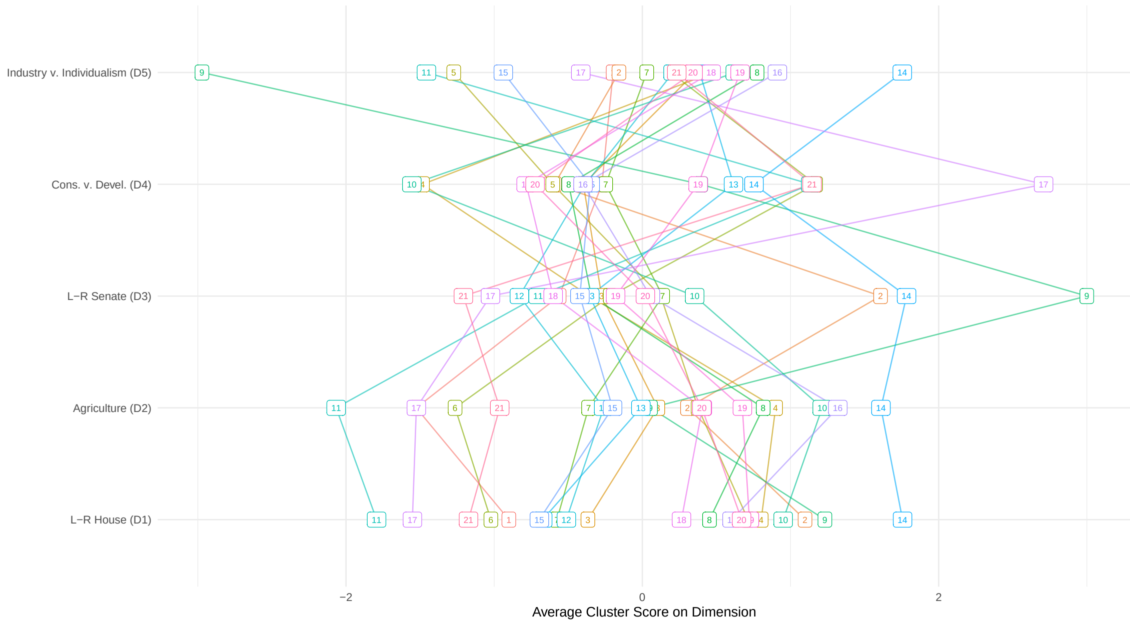

Figure 7. Cluster Heterogeneity in (Relative) Average Ideal Points Across Dimensions

five dimensions of conflict in the United States, estimating ideal points for over 800 organized interest groups as well as over 900 members of Congress serving between the 109th to 114th Congresses across those dimensions. In addition to two separate Left-Right dimensions for the House and Senate, we also observe an Agriculture dimension, along with dimensions emphasizing questions of Conservation v. Development which often pit environmental interests against specific industries but not peak business organizations as a whole. Finally, we observe a dimension that pits interests seeking collective Industrial progress against those emphasizing Individualistic rights and liberties.

Together, an organization's ideal points across these dimensions forms its multidimensional ideal point estimate (mIGscore). Cluster analyses applied to mIGscores identify 21 distinct "factions" in American politics—that is, groups of organizations that exhibit similar preferences across the five dimensions—and that exhibit interesting differences in their size as well as their absolute and relative emphasis on public position-taking, lobbying, and campaign contributions as influence strategies.

We aim to continue build upon these findings in several key ways. First, our substantive interpretation of the dimensions we have identified here are provisional, and may be updated as we conduct case studies of the politics surrounding individual bills that load strongest on each dimension and the interest group politics that surrounded them; indeed, preliminary analysis of the bills loading strongly onto Dimension 5 (what we're currently calling "Industry vs. Individualism") suggest it might be better characterized as a "tech vs. privacy" dimension. Second, the organization-level analyses in this paper are confined to those organizations that happen to be among the closest to their cluster's centroid. Many important organizations—e.g., the American Medical Association, the AARP, the NAACP, the NRA, and major technology companies—do not have this feature and so have not yet been discussed here. We plan to examine these and other "odd fit" organizations, particularly to see if the ability of organizations to "bridge" factions is a potential path, or obstacle, to influence. These analyses can, helpfully, be undertaken with the current dataset.

Perhaps the most significant long-term area of expansion will only become clear if substantial reforms are made to the American electoral system. The 5 dimensions and 21 factions identified here are distinctive in their voting records during a period in which partisan agenda-control is artificially pushing their position-taking *toward undimensionality and party teamsmanship.* Under such conditions, members of a given faction may be functionally tied to one of the two major parties, may serve as a bipartisan "bridge" between them, or may avoid votes (and bills) that cleanly divide the parties. Further work may identify the extent to which members of different factions are similar in how they navigate the current era partisan polarization and competition.

More speculatively, though, is what might happen if the partisan agenda-control, and concomitant "artificial" unidimensionality it promotes, were relaxed. This could be the result of electoral reforms that encourage bipartisan governing approaches or that make third parties more viable contestants for seats and participation in coalition governments. In the long term following such reforms, we could see an explosion of factions, of dimensions, or both.

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Appendix 1. Gibbs Sampler

1. **Step 1 – Augment the Binary Inputs:** For each y_{ij}, draw x_{ij} from a truncated normal distribution according to:

$$x_{i,j} \sim \begin{cases} \mathcal{TN}_{-\infty,0}(\boldsymbol{\lambda}_j \boldsymbol{\omega}_i - \alpha_j, 1) \text{ if } y_{i,j} = 0 \\ \mathcal{TN}_{0,\infty}(\boldsymbol{\lambda}_j \boldsymbol{\omega}_i - \alpha_j, 1) \text{ if } y_{i,j} = 1 \\ \mathcal{N}(\boldsymbol{\lambda}_j \boldsymbol{\omega}_i - \alpha_j, 1) \text{ if } y_{i,j} \text{ is missing} \end{cases} \tag{3}$$

2. **Step 2 – Sample Bill Loadings, $\boldsymbol{\Lambda}$:** Define K^+ as the current number of active features. For each $j \in (1, ..., P)$ and $k \in (1, ..., K^+)$, define the odds that dimension k is active for bill j as:

$$t_{jk} = \frac{P(\lambda_{jk} \neq 0 | \boldsymbol{X}, \boldsymbol{\Omega}, \boldsymbol{\alpha})}{P(\lambda_{jk} = 0 | \boldsymbol{X}, \boldsymbol{\Omega}, \boldsymbol{\alpha})} = \frac{P(\boldsymbol{x}_j^{-k} | \lambda_{jk} \neq 0)}{P(\boldsymbol{x}_j^{-k} | \lambda_{jk} = 0)} \times \frac{P(\lambda_{jk} \neq 0)}{P(\lambda_{jk} = 0)} \tag{4}$$

where $\boldsymbol{x}_j^{-k} = \boldsymbol{x}_j + \alpha_j - \boldsymbol{\Omega}\boldsymbol{\lambda}_j^{-kT}$ and $\boldsymbol{\lambda}_j^{-k}$ the current vector of loadings for bill j with the k^{th} element set to zero.
The first part of this product can be computed as:

$$\frac{P(\boldsymbol{x}_j^{-k} | \lambda_{jk} \neq 0)}{P(\boldsymbol{x}_j^{-k} | \lambda_{jk} = 0)} = \frac{\int \mathcal{N}_N(\boldsymbol{x}_j^{-k} \mid \boldsymbol{\omega}_k \lambda_{jk}^T, \mathcal{I}_N) \mathcal{N}(\lambda_{jk} \mid 0, \gamma_k^{-1}) d\lambda_{jk}}{\mathcal{N}_N(\boldsymbol{x}_j^{-k} \mid \boldsymbol{0}, \mathcal{I}_N)} = \sqrt{\frac{\gamma_k}{\boldsymbol{\omega}_k^T \boldsymbol{\omega}_k + \gamma_k}} \exp\left[\frac{(\boldsymbol{\omega}_k^T \boldsymbol{x}_j^{-k})^2}{2(\boldsymbol{\omega}_k^T \boldsymbol{\omega}_k + \gamma_k)}\right] \tag{5}$$

The second part of this product comes directly from the Indian Buffet Process prior (add references here):

$$\frac{P(\lambda_{jk} \neq 0)}{P(\lambda_{jk} = 0)} = \frac{m_{jk}}{P - m_{jk} + 1} \tag{6}$$

where m_{jk} is the number of elements in column k of $\boldsymbol{\Lambda}$ that are non-zero **setting λ_{jk} equal to zero.**
Using t_{jk}:

$$P(\lambda_{jk} \neq 0 | -) = \frac{t_{jk}}{1 + t_{jk}} \tag{7}$$

Under this Gibbs sampling scheme, set $\lambda_{jk} = 0$ with probability $1 - P(\lambda_{jk} \neq 0|-)$. If $\lambda_{jk} \neq 0$, then draw λ_{jk} from the conditional posterior:

$$\lambda_{jk} \sim \mathcal{N}(\lambda_{jk} \mid (\boldsymbol{\omega}_k^T \boldsymbol{\omega}_k + \gamma_k)^{-1} \boldsymbol{\omega}_k^T \boldsymbol{x}_j^{-k}, (\boldsymbol{\omega}_k^T \boldsymbol{\omega}_k + \gamma_k)^{-1}) \tag{8}$$

After completing the iteration over the entire matrix of loadings, $\boldsymbol{\Lambda}$, each element should rescaled to normalize the posterior variance:

$$\lambda_{j,k} = \frac{\lambda_{j,k}}{\sqrt{1 + \sum_{h=1}^{K^+} \lambda_{j,h}^2}} \tag{9}$$

Finally, fully inactive dimensions (e.g. any k where $\sum_{j=1}^{P} \lambda_{jk} = 0$) should be deleted. If this step is scheduled immediately after the binary augmentation step, then no other deletions are necessary.

3. **Step 3 – Sample Organization Ideal Points, $\boldsymbol{\Omega}$:**
Let $\boldsymbol{\mu}_0$ be a $N \times K^+$ matrix of priors on the mean of the ideal points and let Σ_0 be a $K \times K$ symmetric positive definite prior on the covariance of the ideal points on each dimension. For each $i \in (1, ..., N)$, draw the K^+-vector of ideal points from:

$$\boldsymbol{\omega}_i \sim \mathcal{N}_{K^+}(\boldsymbol{\omega}_i \mid (\boldsymbol{\Lambda}^T \boldsymbol{\Lambda} + \Sigma_0^{-1})^{-1} (\boldsymbol{\Lambda}^T (\boldsymbol{x}_i + \boldsymbol{\alpha}) + \Sigma_0^{-1} \boldsymbol{\mu}_{i0}), (\boldsymbol{\Lambda}^T \boldsymbol{\Lambda} + \Sigma_0^{-1})^{-1}) \tag{10}$$

Most often, **specific priors should not be placed on the ideal points and the default prior with mean 0 and variance 1 should be used**! Because the stochastic search over the dimensionality of the latent space is random, there is little reason that most dimensions should be associated with prior beliefs. However, there are some specific use-cases where a prior may be desirable on the first dimension. Implement the prior with care!

4. **Step 4 – Sample Bill Loading Precisions, γ_k:**
For each $k \in (1, ..., K^+)$, sample γ_k from:

$$P(\gamma_k|-) \sim \text{Gamma}\left(1 + \frac{m_k}{2}, 1 + \sum_{j=1}^{p} \lambda_{j,k}^2\right) \tag{11}$$

where m_k is the number of elements in column k of $\mathbf{\Lambda}$ that are not equal to zero.

5. **Step 5 – Sample Bill-level Intercepts, α_j:**
For each $j \in (1, ..., P)$, sample α_j from:

$$P(\alpha_j \mid -) \sim \mathcal{N}\left(\alpha_j \mid \bar{\mu}_j, \frac{1}{N^2}\sum_{i=1}^{N}(\mu_{i,j} - \bar{\mu}_j)^2\right) \tag{12}$$

where $\mu_{i,j} = \boldsymbol{\lambda}_j' \boldsymbol{\omega}_i - x_{i,j}$ and $\bar{\mu}_j = \frac{1}{N}\sum_{i=1}^{N} \mu_{i,j}$

6. **Step 6 – Potentially Increase K^+:**
While dimensions are consistently removed when there are no active bills, dimensions must also be added in order to fully explore the dimensionality of the latent space. This is achieved by adaptively altering the number of active columns in the loadings and ideal point matrices. As shown in (add ref to Roberts and Rosenthal and Bhattacharya and Dunson), an adaptive Gibbs sampler that adds dimensions at irregular but diminishing intervals converges to the posterior mode for the number of dimensions as the number of Gibbs sampling iterations approaches ∞. We adopt the adaptation schedule proposed by (ref to Roberts and Rosenthal).
Let G be the total number of Gibbs sampling iterations to be run and let the Markov Chain be at iteration g. With $P(g) = \exp[\tau_0 + \tau_1 g]$, add a dimension. For $\mathbf{\Lambda}$, $\mathbf{\Omega}$, and $\boldsymbol{\gamma}$, draw P, N, and 1 new values from the priors, respectively. Some constraints can be added to try to add zeroes to the new values of $\boldsymbol{\lambda}_{K^++1}$, but simulations shows that introducing fully saturated dimensions improves mixing in thinned intervals after the more volatile burn-in period.
τ_0 and τ_1 are tuning parameters that must be less than zero. These control the rate at which new dimensions are added and the volatility of the search as $g \to G$. Simulations show that there is little difference in the convergence properties for reasonable choices of these parameters. Good default choices are $\tau_0 = -2.5$ and $\tau_1 = -1 \times 10^{-5}$. However, pilot runs of the algorithm should be done to determine good values - setting τ_0 and/or τ_1 too large leads to unnecessarily long compute times and poor convergence while setting them to be too small leads the algorithm to miss important dimensions that may be uncoverable once all other parameters have converged to their stationary states. As always, running the Gibbs sampler multiple times with multiple different starting values and learning rates is the best way to ensure that the MCMC procedure has converged to meaningful values.