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Choosing Size of Government Under Ambiguity: Infrastructure Spending and Income Taxation

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Abstract

Attempting to shed light on the optimal size of government, economists have analyzed planning problems that specify a set of feasible taxation-spending policies and a social welfare function. The analysis characterizes the optimal policy choice of a planner who knows the welfare achieved by each policy. This paper examines choice of size of government by a planner who has partial knowledge of population preferences and the productivity of spending. This is a problem of decision making under ambiguity. Focusing on income-tax-financed public spending for infrastructure that aims to enhance productivity, Manski examines scenarios where the planner observes the outcome of a status quo policy and uses various decision criteria (i.e., expected welfare, maximin, Hurwicz, minimax-regret) to choose policy. The analysis shows that the planner can reasonably choose a wide range of spending levels—thus, a society can rationalize having a small or large government. He concludes that to achieve credible conclusions about the desirable size of government, researchers need to vastly improve current knowledge of population preferences and the productivity of public spending.

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"Uncertainty is not something which should be considered as a theoretically interesting refinement or extension of standard theory and methodology, but a central factor of eminently practical importance. Sometimes uncertainty is itself the heart of the matter when decisions are to be taken." Johansen (1978, pp. 263-264.

1. Introduction

The optimal size of government has been a subject of continuing debate. Disagreements may stem in part from the fact that "size of government" is an imprecise term—persons using it may not interpret it the same way. Persons who share a common understanding of the term may disagree on what size is optimal. They may have different normative perspectives on social welfare or different beliefs about the outcomes yielded by alternative policy choices.

Attempting to shed light on the optimal size of government, economists have posed and analyzed social planning problems. A standard exercise specifies a set of feasible policies and a social welfare function, typically utilitarian. The planner is assumed to know the welfare achieved by each policy. The analysis characterizes the optimal policy.

One prominent body of research, stimulated by Mirrlees (1971), has considered the use of income taxation to redistribute income, given fixed public spending. Economists have derived optimal tax schedules under the assumption that the planner knows the income-leisure preferences of the population. Another, following Barro (1990), has considered the use of public spending to promote growth. Economists have derived optimal tax-financed spending levels under the assumption that the planner knows the consumption preferences of the population and how public spending affects aggregate output. Among the simplifying assumptions that Mirrlees (1971) acknowledged in his introductory section, he stated (p. 176): "The State is supposed to have perfect information about the individuals in the economy, their utilities and, consequently, their actions." This assumption has been standard throughout the subsequent literature on optimal taxation and also in the literature relating public spending to growth.

In practice, lack of knowledge of the welfare achieved by alternative policies severely limits the relevance of the standard exercise to actual policy choice. In recent work, I have shown that credible revealed preference analysis of time allocation reveals only a bit about the income-leisure preferences of the population and I have argued that strong conclusions about income tax policy drawn from conventional econometric analysis rest on untenable assumptions (Manski, 2012). Analysis of policies that use taxation to finance public spending is even more challenging, requiring inference on both consumer preferences and the productivity of public spending. We know little about either.

In a research program that began with Manski (2000), I have urged that economists studying social planning maintain credible assumptions and view planning as a problem of policy choice under ambiguity rather than as a deterministic optimization problem. In various contexts, I have shown how basic principles of decision theory may be applied to make reasonable policy choices with partial knowledge of policy outcomes. See Manski (2005, 2006, 2007, 2009, 2010, 2011a). Work with related themes but studying different planning problems has been performed by researchers studying macroeconomic planning under uncertainty (Johansen, 1978) and robust macroeconomic policy (Hansen and Sargent, 2008; Barlevy, 2011).

This paper examines choice of size of government as a problem of planning under ambiguity. Among the many important aspects of "size of government," I focus on tax-financed public spending for infrastructure that aims to enhance private productivity. Even as strong a proponent of private enterprise as Milton Friedman recognized the need for government to provide infrastructure for voluntary exchange, writing (Friedman, 1955): "In . . . a free private enterprise exchange economy, government's primary role is to preserve the rules of the game by enforcing contracts, preventing coercion, and keeping markets free." This statement focuses on the need for government to provide laws, regulations, and a justice system to enforce them. One might reasonably add many further governmental functions, including formation and execution of monetary policy, provision and oversight of transportation and communications, protection of the environment, and conduct or support of research. Each of these and other functions may be performed with varying intensity, at correspondingly varying cost. Taxation is the main mechanism that governments use to finance infrastructure.

My focus on infrastructure spending is similar to that of the growth literature exemplified by Barro (1990). However, my formalization of the planning problem stems from the one used by Mirrlees (1971) to study optimal income taxation. Following Mirrlees, I pose in Section 2 a static setting in which each member of a population allocates time to paid work and to the various non-paid activities that economists have traditionally called leisure. Persons have predetermined heterogeneous wages. An income tax schedule is applied to gross income, yielding net income. Persons allocate time to maximize utility, which increases with net income and leisure. Social welfare is utilitarian.

I depart from the Mirrlees setup in three main ways. First, government chooses how much to spend on infrastructure and on activities that directly affect personal utility. Second, persons may have heterogeneous preferences for income, leisure, and public spending. Third, the planner may have partial knowledge of population preferences and of the productivity of infrastructure spending. In contrast, research on optimal income taxation has studied the use of taxation to redistribute income while holding government spending fixed and has assumed that all persons have the same, known, preferences.

My formalization differs more substantially from that used by growth theorists. They typically pose a dynamic setup that permits no heterogeneity of any kind within the population. The planner is assumed to know the consumption preferences of the representative agent. This agent is depicted as an independent entrepreneur whose output may be affected by public spending.

My formalization also differs substantially from that of the "new dynamic public finance." Research on this subject extends the Mirrlees setup to dynamic settings in which heterogeneous persons make time-allocation and savings decisions under uncertainty. See, for example, Kotcherlakota (2010). This literature assumes that the planner and the members of the population have rational expectations. Thus, it does not engage the problem of planning under ambiguity.

My central concern is partial knowledge of population preferences and the productivity of spending.

Partial knowledge generates two distinct difficulties. First, the planner may be unable to predict tax revenue with certitude and, thus, may not know if a policy will yield a balanced budget. Second, he may be unable to determine the welfare achieved by a policy. I bypass the first issue and focus on the second.

The first issue is quite difficult to address in generality. Satisfactory evaluation of policies that may not yield balanced budgets requires specification and analysis of a dynamic planning problem that permits surpluses and deficits to occur and recognizes their intertemporal welfare implications. This poses a larger challenge than I feel able to confront. To bypass the complexity of dynamic policy evaluation under ambiguity, I consider settings in which the planner can ensure budget balance by choosing components of policies sequentially rather than simultaneously. The idea is to first choose policy components that suffice to determine tax revenue and then use knowledge of tax revenue to choose remaining components so as to balance the budget. This idea can be implemented in certain settings.

When budget balance can be ensured by sequential choice of policy components, planning may be readily studied using established criteria for static decision making under ambiguity. I do so in two settings.

Section 3 makes relatively weak preference assumptions that serve well to discuss general principles for planning under ambiguity and to juxtapose various decision criteria. I assume that public spending is known to have two scalar components, one being spending on infrastructure and the other on what I will call *amenities*. Infrastructure spending affects wages and non-labor income but does not directly affect personal utility. Amenities spending affects personal utility but does not affect wages or non-labor income. I assume that utility is additively separable in spending on amenities. These assumptions enable the planner to achieve budget balance by first choosing a tax schedule and infrastructure spending, then observing the time allocation decisions of the population and the resulting tax revenue, and finally choosing amenities spending to balance the budget.

Section 4 makes much stronger assumptions that yield simple closed-form findings. First, the planner only considers tax schedules that make the tax proportional to income. Second, persons have Cobb-

Douglas income-leisure preferences and no non-labor income. These assumptions imply that time-allocation choices are invariant to policy and they enable the planner to achieve budget balance by first choosing the level of public spending, then observing the resulting tax revenue, and finally choosing the tax rate to balance the budget. Third, for further simplicity, I assume that all public spending is on infrastructure and that wages are person-specific positive constants multiplied by an aggregate production function expressing the wage-enhancing effect of spending on infrastructure. Finally, the planner has partial knowledge of the aggregate production function, obtained by observing the outcome of a status quo policy and by assuming that public spending enhances wages but with diminishing marginal returns. Then the space of possible states of nature indexes all concave-monotone aggregate production functions that yield the outcome of the status quo policy.

In this setting, I show that the planner can reasonably choose a wide range of spending levels. Thus, a society can rationalize having a small or large government. The choice made depends on the decision criterion that the planner uses to cope with ambiguity. I consider planning that maximizes subjective expected welfare or that uses one of several criteria—the maximin, minimax-regret, or Hurwicz criterion—that do not place a subjective probability distribution on unknown quantities. In addition to studying choice among a broad set of spending levels, I also examine constrained settings in which the planner is only permitted to make marginal changes from the status quo.

Section 5 draws conclusions that are methodologically constructive and substantively cautionary. The methodologically constructive conclusion is that, when performing normative research on size of government, economists need not study optimization problems whose solution requires far more knowledge than researchers can credibly assert. Decision theory provides a suitable formal framework for study of planning under ambiguity. The analysis of Section 3 shows how to apply this framework in principle and Section 4 shows that it yields simple findings in at least some illustrative settings.

The substantively cautionary conclusion is that study of planning with credible assumptions shows that a wide range of policy choices can be rationalized. The only way to achieve credible conclusions about the desirable size of government is to vastly improve current knowledge of population preferences and the productivity of public spending. There is no immediate way to achieve this, but a research program with a suitably long-run perspective may make progress possible.

Before proceeding, I should alert readers that some prominent concerns of the normative literature on taxation make no appearance in this paper. Economists often study concepts suffused with negative rhetoric, including the "inefficiency," "deadweight loss," and "distortion" of taxation. For example, a recent review article of Saez, Slemrod, and Giertz (2012) states (p.41):

> "under some assumptions *all* responses to taxation are symptomatic of deadweight loss. Taxes trigger a host of behavioral responses intended to minimize the burden on the individual. In the absence of externalities or other market failure, and putting aside income effects, all such responses are sources of inefficiency."

This and many similar statements abstract from the standard rationales for taxation, from redistribution to finance of public spending to treatment of market failures. It is not surprising that taxation should appear negative when no rationale for it is considered.

I think it important to jointly recognize the potential benefits and costs of taxation. Specification of a social welfare function and study of the welfare achieved by alternative policies makes both explicit. Posing an optimal taxation and spending problem appropriately recognizes the benefits and costs of taxation, making it unnecessary to study concepts such as inefficiency and distortion. Indeed, Mirrlees (1971) made no mention of these concepts in his seminal study of optimal income taxation. Although this paper supposes that the planner has less knowledge than was assumed by Mirrlees, it fully endorses his idea that the social welfare function is the only normative concept required for evaluation of policy.

2. Public Spending Financed by Income Taxation

2.1. Spending, Taxation, and Time Allocation

I adapt the classical static model of time allocation in which persons allocate one unit of time to work and leisure in an environment with predetermined income taxation and public spending. This model is obviously simplistic in numerous respects, but it serves well to illuminate core issues regarding planning under ambiguity.

To begin, let J denote a population of persons, formalized as a probability space (J, Ω, P). Each person $j \in J$ has one unit of time, which he must allocate between leisure and paid work. Specifically, j chooses a leisure value λ from a set $\Lambda_j \subset [0, 1]$ of feasible leisure alternatives. Much analysis of time allocation supposes that $\Lambda_j = [0, 1]$. However, it may be more realistic to suppose that only a few allocations are feasible. For example, $\Lambda_j = \{0, ½, 1\}$ means that the feasible options are full-time work ($\lambda = 0$), half-time work ($\lambda = ½$), and no work ($\lambda = 1$).

A policy pairs an income-tax schedule with a public-spending bundle. Let φ denote a policy. Let T_φ denote the tax schedule specified by this policy and let g_φ denote its spending bundle. Then policy φ is fully characterized by the pair (T_φ, g_φ).

Under policy φ, person j receives gross wage $w_j(\varphi)$ for full-time work and receives gross non-labor income $z_j(\varphi)$ from savings or other sources. The fact that wage and non-labor income are subscripted by j and are functions of φ in principle permits them to vary in any manner across the population and across policies. However, to focus attention on the productivity of infrastructure spending, I will assume that policy may affect gross wages and non-labor income only through spending, not through the tax schedule. Therefore, I henceforth write $w_j(g_\varphi)$ and $z_j(g_\varphi)$ rather than $w_j(\varphi)$ and $z_j(\varphi)$. Specifying wage and non-labor income as functions of public spending formalizes the first way that the present analysis differs from the norm in research on optimal income taxation. The standard practice has been to hold public spending fixed and compare policies with different tax schedules.

Although I suppose that the tax schedule does not affect gross income, it does affect net income. If person j allocates time λ to leisure and $1 - \lambda$ to work, he receives gross income $w_j(g_\varphi)(1 - \lambda) + z_j(g_\varphi)$. The tax schedule subtracts tax $T_\varphi[w_j(g_\varphi)\cdot(1 - \lambda) + z_j(g_\varphi)]$, leaving him with net income

$$(1) \qquad y_{\varphi j}(\lambda) \equiv w_j(g_\varphi)(1 - \lambda) + z_j(g_\varphi) - T_\varphi[w_j(g_\varphi)\cdot(1 - \lambda) + z_j(g_\varphi)].$$

Preferences are expressed in person j's utility function $U_j(\cdot, \cdot, \cdot)$, whose arguments are (net income, leisure, public spending). Utility is strictly increasing in income and leisure. Let $\lambda_{\varphi j} \in \Lambda_j$ denote the leisure that j chooses under policy φ. Utility maximization implies that

$$(2) \quad U_j[y_{\varphi j}(\lambda_{\varphi j}), \lambda_{\varphi j}, g_\varphi] \geq U_j[y_{\varphi j}(\lambda), \lambda, g_\varphi], \text{ all } \lambda \in \Lambda_j.$$

Permitting preferences to vary across the population formalizes the second way that the present analysis differs from the norm in research on optimal income taxation, which has assumed that preferences are homogeneous.

Observe that policy can affect the population distribution of net income in three ways. First, as emphasized in research on optimal income taxation, policy need not make the tax proportional to gross income. Second, public spending need not proportionally affect the wages and non-labor income of different persons. Third, if preferences are heterogeneous, policy may differentially affect time-allocation decisions across the population. Thus, the overall impact of policy on the distribution of net income depends jointly on the tax schedule, the nature of public spending, and the time-allocation decisions that persons make in response to policy.

2.2. Utilitarian Welfare Analysis

Utilitarian welfare analysis specifies a social welfare function $W[U_j(y_{\varphi j}, \lambda_{\varphi j}, g_\varphi), j \in J]$. Being utilitarian, the function is increasing in each U_j argument. The specific form of W expresses how society weights the interests of different persons. The usual practice is to sum selected cardinal representations of personal utilities. Doing this and normalizing by the size of the population yields

$$(3) \quad W[U_j(y_{\varphi j}, \lambda_{\varphi j}, g_\varphi), j \in J] \ = \ E[U(y_\varphi, \lambda_\varphi, g_\varphi)] \ \equiv \ \int U_j(y_{\varphi j}, \lambda_{\varphi j}, g_\varphi)dP(j).$$

Thus, social welfare is the mean cardinal utility of the members of the population.

The description of welfare in (3) enables comparison of balanced-budget policies; that is, ones generating tax revenue equal to the cost of public spending. The literatures on optimal taxation and on growth restrict attention to balanced-budget policies. Let $c(g_\varphi)$ be the cost per capita of spending bundle g_φ and let

$$(4) \quad R_\varphi \ \equiv \ E\{T_\varphi[w(g_\varphi)(1 - \lambda_\varphi) + z(g_\varphi)]\}$$

denote tax revenue per capita. A balanced budget requires that $R_\varphi = c(g_\varphi)$. Consider a set Φ of policies, all with balanced budgets. A policy is optimal within Φ if it solves the problem

$$(5) \quad \max_{\varphi \in \Phi} \ E[U(y_\varphi, \lambda_\varphi, g_\varphi)].$$

Research on optimal income taxation has posed maximization problems of form (5) and has sought to characterize the solutions. The norm has been to let Φ be a set of policies that share a common status quo spending bundle, say g_S, but have different tax schedules. Researchers have assumed that, conditional on g_S, all persons have the same known income-leisure preferences, expressed in a common sub-utility function $U(\cdot, \cdot, g_S)$. Persons may have heterogeneous wages and non-labor incomes $[w_j(g_S), z_j(g_S); j \in J]$. It has typically been assumed that the planner does not observe these quantities but does know their distribution $P[w(g_S), z(g_S)]$ in the population.

Knowledge of $U(\cdot, \cdot, g_S)$ and $P[w(g_S), z(g_S)]$ suffices to determine the distribution $P(y_\varphi, \lambda_\varphi)$ of net income and leisure that would result under the tax schedule of each policy $\varphi \in \Phi$. This in turn enables determination of the social welfare $E[U(y_\varphi, \lambda_\varphi, g_S)]$ achieved by each policy. Hence, this knowledge enables determination of the optimal policy.

2.3. Planning under Ambiguity

My concern is planning when public spending may vary across policies and the planner has insufficient knowledge to determine the optimal policy. Permitting public spending to vary across policies may complicate optimization problem (5) in practice, but the problem remains solvable in principle. Partial knowledge of population preferences and the productivity of public spending may create a more fundamental difficulty. It may make it logically impossible to determine the optimal policy. Then the planner faces a problem of policy choice under ambiguity.

Partial knowledge of preferences and productivity raises two distinct issues. First, the planner may be unable to predict tax revenue with certitude and, thus, may not know if a policy will yield a balanced budget. Second, he may be unable to determine the welfare achieved by a policy. I will bypass the first issue and focus on the second.

To satisfactorily evaluate policies that may not yield balanced budgets requires specification and analysis of a dynamic planning problem that permits budget surpluses and deficits to occur and recognizes their intertemporal welfare implications. However, study of dynamic choice of size of government poses a larger challenge than I feel able to confront here. The dynamic models used in research relating public spending to growth provide no foundation because they have assumed that the planner has perfect foresight and thus is able to balance the budget.

To bypass the complexity of dynamic policy evaluation under ambiguity, I will consider settings in which the planner can ensure budget balance by choosing components of policies sequentially rather than simultaneously. The idea is to first choose policy components that suffice to determine tax revenue and then use knowledge of tax revenue to choose remaining components so as to balance the budget. This idea can be implemented in certain settings. When implementation is possible, policy choice may be studied using established criteria for static decision making under ambiguity.

Sections 3 and 4 study such sequential policy choice in two settings. Section 3 makes relatively weak assumptions that serve well to discuss general principles for planning under ambiguity. Section 4 makes much stronger assumptions that yield interpretable closed-form findings.

3. Sequential Policy Choice with Preferences Separable in Amenities Spending

3.1. The Setting

In this section I assume that public spending has two scalar components, one on infrastructure and the other on amenities. The infrastructure component, g_1, may affect wages and non-labor income, but it does not directly affect personal utility. The amenities component, g_2, may affect personal utility in an additively separable manner, but it does not affect wages or non-labor income. Aggregation of the many disparate forms of public spending into these two aggregated components is a huge simplification whose payoff is that it enables achievement of budget balance.

Formally, the assumptions suppose that

(6) $g = (g_1, g_2)$; $c(g) = g_1 + g_2$; $w_j(g) = w_j(g_1)$; $z_j(g) = z_j(g_1)$; $U_j(y, \lambda, g) = U_{1j}(y, \lambda) + U_{2j}(g_2)$.

In (6) I continue to use the notation $w_j(\cdot)$ and $z_j(\cdot)$ to express wage and non-labor income as functions of g_1 alone. I introduce new notation U_{1j} and U_{2j} for the sub-utility functions of (y, λ) and g_2 respectively.

Given (6), I suppose that the planner chooses a *feasible* policy φ that achieves budget balance sequentially, first by specifying $(T_\varphi, g_{1\varphi})$, then by observation of tax revenue, and finally by choosing $g_{2\varphi}$ to balance the budget. That is,

(i) The government chooses a tax schedule T_φ and infrastructure spending $g_{1\varphi}$.
(ii) Persons choose time allocation given knowledge of $(T_\varphi, g_{1\varphi})$. Given that preferences are separable in $g_{2\varphi}$, persons can optimize time allocation without knowledge of $g_{2\varphi}$. The productivity of infrastructure spending and the time-allocation decisions of the population determine tax revenue R_φ.
(iii) The government observes R_φ and sets $g_{2\varphi}$ to balance the budget. Thus, $g_{2\varphi} = R_\varphi - g_{1\varphi}$.

I define policy φ to be feasible if the planner knows enough about population preferences and the productivity of public spending to be certain that $R_\varphi - g_{1\varphi} \geq 0$. Thus, the planner knows enough to be certain ex ante that a non-negative value of $g_{2\varphi}$ will yield budget balance ex post.

3.2. Criteria for Policy Choice under Ambiguity

With this background, we can consider policy choice under ambiguity. Let Φ be a specified set of feasible policies. Given the assumptions in (6), each $\varphi \in \Phi$ yields welfare

$$(7) \quad E[U(y_\varphi, \lambda_\varphi, g_\varphi)] \;=\; E[U_1(y_\varphi, \lambda_\varphi)] + E[U_2(R_\varphi - g_{1\varphi})].$$

The planner's problem is to choose a policy in Φ given partial knowledge of population preferences and the productivity of infrastructure spending. To address the problem, I apply standard principles of decision theory.

To begin, the planner specifies a *state space*, say Γ, indexing all combinations of population preferences and spending productivity that he deems possible. For each state $\gamma \in \Gamma$ and policy $\varphi \in \Phi$, let

$$(8) \quad E_\gamma[U(y_\varphi, \lambda_\varphi, g_\varphi)] \;=\; E_\gamma[U_1(y_\varphi, \lambda_\varphi)] + E_\gamma[U_2(R_\varphi - g_{1\varphi})]$$

denote the welfare of φ in state γ.

Specification of the state space enables the planner to eliminate dominated policies from consideration. Policy φ is *weakly dominated* if there exists another feasible policy φ' that yields weakly larger welfare in every possible state of nature and strictly larger welfare in some state. Thus, φ is weakly dominated if $E_\gamma[U(y_\varphi, \lambda_\varphi, g_\varphi)] \le E_\gamma[U(y_{\varphi'}, \lambda_{\varphi'}, g_{\varphi'})]$ for all $\gamma \in \Gamma$ and $E_\gamma[U(y_\varphi, \lambda_\varphi, g_\varphi)] < E_\gamma[U(y_{\varphi'}, \lambda_{\varphi'}, g_{\varphi'})]$ for some $\gamma \in \Gamma$. Elimination of weakly dominated policies is normatively compelling. If φ is weakly dominated by φ', the planner is certain that φ' yields at least the welfare of φ and deems it possible that φ' yields greater welfare.

The hard part of planning under ambiguity is choice among undominated policies. If two policies are undominated, then either both yield the same welfare in all possible states or there exist distinct states in which each yields strictly higher welfare than the other. In the former case, both policies are equally good choices and the decision maker is indifferent between them. In the latter case, the planner cannot definitively order the policies. Each may possibly yield higher welfare than the other. Thus, the normative question "How should the planner choose between these policies?" has no unambiguously correct answer.

Although there is no uniquely optimal choice among undominated actions, decision theorists have not wanted to abandon the idea of optimization. So they have proposed various ways of transforming the unknown welfare function into one that can be maximized. The most familiar idea to economists is to average the elements of Γ and maximize the resulting function. This yields maximization of expected welfare. In the present setting, the expected welfare criterion solves the optimization problem

$$(9) \qquad \max_{\varphi \in \Phi} \int E_{\gamma}[U(y_{\varphi}, \lambda_{\varphi}, g_{\varphi})]d\pi,$$

where π is a specified subjective probability distribution on Γ.

The expected welfare criterion is reasonable when a planner can motivate his choice of π, but specification of a subjective distribution constitutes a form of knowledge that may be lacking in practice. To cope with such situations, decision theorists have suggested and studied a variety of other criteria. A broad idea is to choose a policy that, in some well-defined sense, works uniformly well across the entire state space. This idea has been formalized in various ways.

The maximin criterion evaluates a policy by the minimum welfare that it may possibly yield. In the present setting, the criterion solves the problem

$$(10) \qquad \max_{\varphi \in \Phi} \ \min_{\gamma \in \Gamma} \ E_{\gamma}[U(y_{\varphi}, \lambda_{\varphi}, g_{\varphi})].$$

An extension of the maximin idea, called the Hurwicz criterion, evaluates policies by an average of the minimum and maximum welfare that they may possibly yield. The criterion solves the problem

$$(11)\qquad \max_{\varphi \in \Phi}\ \alpha \{\max_{\gamma \in \Gamma}\ E_\gamma[U(y_\varphi, \lambda_\varphi, g_\varphi)]\} + (1 - \alpha)\{\min_{\gamma \in \Gamma} E_\gamma[U(y_\varphi, \lambda_\varphi, g_\varphi)]\},$$

where α and 1 − α are the weights given to maximum and minimum welfare respectively. Finally, the minimax-regret criterion evaluates a policy by its maximum regret: the regret of a policy in a particular state of nature is the loss in welfare that would occur if one were to choose this policy rather than the one that is best in this state. This criterion thus solves the problem

$$(12)\qquad \min_{\varphi \in \Phi}\ \max_{\gamma \in \Gamma}\ [\max_{\varphi' \in \Phi} U(y_{\varphi'}, \lambda_{\varphi'}, g_{\varphi'}) - U(y_\varphi, \lambda_\varphi, g_\varphi)].$$

3.3. Discussion

I wrote above that the normative question "How should the planner choose?" under ambiguity has no unambiguously correct answer. I have consequently posed a set of decision criteria, each of which may be viewed as reasonable. In a series of studies of specific decision problems, I have found it revealing to compare the choices yielded by these criteria. See Manski (2004, 2006, 2009, 2010) and Brock and Manski (2011). The setting of this section is too abstract to meaningfully compare the criteria, but I will in Section 4 examine a simpler setting that yields readily to analysis.

Beginning with Savage (1954), some decision theorists have argued that a decision maker not only *might* but *should* place a subjective distribution on the states of nature and maximize expected welfare. Savage famously proved a representation theorem showing that maximization of subjective expected welfare is mathematically equivalent to behaving in accord with a set of consistency axioms on hypothetical choice behavior. He felt that adherence to the axioms constitutes the essence of rational choice with partial knowledge. Hence, he drew the normative conclusion that a decision maker should place a subjective distribution on unknowns and should maximize expected welfare.

Other decision theorists have not agreed that adherence to the Savage axioms constitutes the essence of rational choice. There are multiple schools of thought on the matter. A diverse body of modern axiomatic decision theory studies various systems of axioms on hypothetical choice behavior. Some of these axiom systems do not imply that a person will place a subjective distribution on unknowns, never mind maximize expected welfare. See Binmore (2009) for a recent perspective. I personally have gone further and have argued that the basic concern of axiomatic decision theory, characterization of consistency in behavior across hypothetical choice scenarios, is not relevant to actual decision making (Manski, 2011b).

To find the present paper of potential interest, a reader need not subscribe to a particular viewpoint on axiomatic decision theory. He or she need only find it reasonable to conceive of a planner who lacks a subjective distribution on the states of nature. This planner must still somehow cope with ambiguity.

4. Sequential Policy Choice with Proportional Taxes and Cobb-Douglas Preferences

The literature on optimal income taxation with fixed public spending and complete knowledge of income-leisure preferences has long recognized that optimization problem (5) can be hard to solve in practice even when it is solvable in principle. As a consequence, researchers have focused on simple special cases that yield to analysis.

Planning under ambiguity compounds the practical difficulty. Each of the decision criteria presented in Section 3.2 poses an optimization problem that is more complex than those studied in research on optimal income taxation. Hence, I similarly find it helpful to focus on a tractable special case. Section 4.1 poses the setting to be examined. Section 4.2 characterizes optimal policy choice. Sections 4.3 through 4.5 consider planning under ambiguity.

4.1. The Setting

To begin, I assume that the planner only considers tax schedules that make the tax proportional to gross income. Thus, each policy φ under consideration has

$$(13) \quad T_\varphi[w_j(g_\varphi)(1-\lambda_{j\varphi}) + z_j(g_\varphi)]\} \;=\; t_\varphi\cdot[w_j(g_\varphi)(1-\lambda_{j\varphi}) + z_j(g_\varphi)],$$

where $t_\varphi \in [0, 1)$ is the proportional tax rate. It follows that policy φ yields a balanced budget if

$$(14) \quad t_\varphi I(g_\varphi) \;=\; c(g_\varphi),$$

where $I(g_\varphi) \equiv E[w(g_\varphi)(1-\lambda_\varphi) + z(g_\varphi)]$ is mean gross income in the population.

In general, mean income is an implicit function of the tax rate because taxes affect time-allocation decisions. However, time allocation is invariant with respect to the tax rate if persons have Cobb-Douglas income-leisure preferences and no non-labor income. I now specialize assumption (6) to impose these restrictions. I also, for further simplicity, assume that all public spending is on infrastructure and that wages are person-specific positive constants multiplied by an aggregate production function expressing the wage-enhancing effect of infrastructure spending.

Formally, I henceforth assume that

$$(15) \quad g = g_1; \quad c(g) = g_1; \quad w_j(g) = \theta_j f(g_1); \quad z_j(g_1) = 0; \quad U_j(y, \lambda, g) = \delta_j\cdot\log(y) + (1-\delta_j)\log(\lambda);$$
$$(\theta_j > 0,\ 0 < \delta_j < 1,\ \text{all } j \in J); \quad f(g_1) > 0,\ \text{all } g_1 > 0.$$

Here θ_j is the person-specific determinant of wage (say ability for short) and $f(\cdot)$ is the aggregate function of spending that multiplies it. I use the logarithmic cardinal representation of Cobb-Douglas preferences, with person-specific preference weights (δ_j, $1 - \delta_j$) on log (income) and log (leisure). Given that all public spending is on infrastructure, I henceforth shorten the notation by dropping the subscript on g_1; thus, g_1 is now called g.

These assumptions imply that individual time allocation is policy-invariant, the optimal leisure for person j being $\lambda_{j\varphi} = 1 - \delta_j$ for all $t_\varphi \in [0, 1)$ and $g_\varphi > 0$. Indeed, j's time allocation is $1 - \delta_j$ even if the person does not know what policy the government will choose. Hence, mean income reduces to

$$\text{(16)} \qquad I(g_\varphi) = f(g_\varphi)\cdot E(\theta\delta).$$

Thus, mean income is determined multiplicatively by infrastructure spending through $f(g_\varphi)$ and by population composition through $E(\theta\delta)$, which expresses the covariation of ability (θ) and preference for income (δ).

In this setting, a planner can choose a feasible policy that achieves budget balance sequentially. Persons select time allocations that do not depend on the chosen policy. The planner first chooses public spending and observes the resulting mean income, which depends on the productivity of spending. He then chooses the tax rate to balance the budget. Policy φ is feasible if the planner is certain ex ante that it will yield sufficient income to more than cover the cost of government spending; that is, $I(g_\varphi) > g_\varphi$. If so, the planner can balance the budget by setting the tax rate $t_\varphi = g_\varphi/I(g_\varphi)$ ex post.

Observe that the assumptions made here imply that policy is neutral with respect to the population distribution of net income. The net income of person j under policy φ is $(1 - t_\varphi)\cdot f(g_\varphi)\cdot\theta_j\delta_j$. Thus, net income is the policy-specific quantity $(1 - t_\varphi)f(g_\varphi)$ times the person-specific quantity $\theta_j\delta_j$.

4.2. Optimal Policy Choice

With this background, I consider optimal policy choice and then planning under ambiguity. Let Φ be a specified set of feasible policies. Given the maintained assumptions and the equation for budget balance, each $\varphi \in \Phi$ yields welfare

$$
\begin{aligned}
(17)\quad E[U(y_\varphi, \lambda_\varphi, g_\varphi)] &= E[\delta \cdot \log(y_\varphi)] + E[(1-\delta)\log(\lambda_\varphi)] \\
&= E[\delta \cdot \log[(1-t_\varphi) \cdot \theta \cdot f(g_\varphi) \cdot \delta] + E(1-\delta) \cdot \log(1-\delta) \\
&= E(\delta) \cdot \log(1-t_\varphi) + E(\delta)\{\log[f(g_\varphi)]\} + E[\delta \cdot \log(\theta\delta)] + E(1-\delta) \cdot \log(1-\delta) \\
&= E(\delta) \cdot \log[1 - g_\varphi / I(g_\varphi)] + E(\delta)\{\log[f(g_\varphi)]\} + E[\delta \cdot \log(\theta\delta)] + E(1-\delta) \cdot \log(1-\delta) \\
&= E(\delta) \cdot \log[I(g_\varphi) - g_\varphi] + K,
\end{aligned}
$$

where $K \equiv -E(\delta) \cdot \log[E(\theta\delta)] + E[\delta \cdot \log(\theta\delta)] + E(1-\delta) \cdot \log(1-\delta)$ collects several policy-invariant terms. Dropping multiplicative and additive constants does not affect the optimal policy nor the choices made under any of the criteria for decision making under ambiguity considered in this paper. This done, it follows from (17) that optimization problem (5) reduces to

$$
(18)\qquad \max_{\varphi \in \Phi} \quad \log[I(g_\varphi) - g_\varphi].
$$

The optimal policy is thus easy to determine if one knows the function $I(\cdot)$ expressing how mean gross income varies with public spending.

4.3. Partial Knowledge of the Productivity of Public Spending

To go beyond abstract discussion of planning under ambiguity requires consideration of particular informational settings. In studies of other planning problems, I have found it fruitful to consider settings in which the planner observes the outcome of a status quo policy and combines this empirical evidence with credible restrictions on the welfare function (Manski, 2006, 2010). I do so here as well.

In the present setting, the critical unknown quantity is the function $I(\cdot)$ expressing how mean gross income varies with public spending. I will suppose that the planner observes public spending g_S and the mean income $I_S \equiv I(g_S)$ realized under a status quo policy, denoted S. I also suppose that the status quo policy is feasible; thus, $I_S > g_S$.

Observation of (g_S, I_S) reveals one point on function $I(\cdot)$. To extrapolate beyond this empirical evidence, the planner has to know something about the shape of $I(\cdot)$. I will suppose that the planner finds it credible to assume that $I(\cdot)$ is concave-monotone; thus, public spending enhances wages but has diminishing marginal returns. He does not find it credible to assume anything more about $I(\cdot)$.

With this knowledge, the space Γ of possible states of nature indexes all concave-monotone functions $I(\cdot)$ such that $I(g_S) = I_S$. It follows that the feasible policies are those that set $g \in (0, I_S)$. The value $g = g_S$ is feasible by observation. Each $g \in (0, g_S)$ is feasible because, for all $\eta \in (0, 1)$,

$$(19) \quad I(\eta g_S) \geq \eta I_S + (1 - \eta)I(0) \geq \eta I_S > \eta g_S.$$

The first inequality holds by concavity of $I(\cdot)$, the second because $I(0) \geq 0$, and the third because $I_S > g_S$. Each $g \in (g_S, I_S)$ is feasible because monotonicity yields $I(g) \geq I_S$ for $g \geq g_S$. Each $g \geq I_S$ is not feasible because the state space admits the possibility that $I(g) = I_S$ for all $g \geq g_S$. Finally, $g = 0$ is not feasible because the state space admits the possibility that $I(0) = 0$.

[insert Figure 1 here]

Figure 1 shows the possible mean-income functions on the domain $g \in (0, I_S)$. The possible $I(\cdot)$ are the concave-monotone functions that pass through the shaded areas of the figure. The linear function $I(g) = (I_S/g_S)\cdot g$, with slope $I_S/g_S > 1$, gives the lower bound on I(g) for $g \le g_S$ and the upper bound for $g \ge g_S$. The flat function $I(g) = I_S$ gives the upper bound on I(g) for $g \le g_S$ and the lower bound for $g \ge g_S$. Thus, for each $g \in (0, I_S)$, we have these sharp bounds on I(g):

$$(20) \quad g \le g_S \Rightarrow (I_S/g_S)g \le I(g) \le I_S,$$
$$g \ge g_S \Rightarrow I_S \le I(g) \le (I_S/g_S)g.$$

Cardinal welfare with spending g is $\log[I(g) - g]$; that is, the logarithm of the vertical distance between I(g) and the 45° dotted line shown in the figure. The sharp bounds on I(g) given in (20) imply these sharp bounds on welfare:

$$(21) \quad g \le g_S \Rightarrow \log(I_S - g_S) + \log(g/g_S) \le \log[I(g) - g] \le \log(I_S - g),$$
$$g \ge g_S \Rightarrow \log(I_S - g) \le \log[I(g) - g] \le \log(I_S - g_S) + \log(g/g_S).$$

Consideration of the bounding functions $I(g) = (I_S/g_S)g$ and $I(g) = I_S$ shows that no feasible spending level is dominated. If the actual mean-income function is $I(g) = (I_S/g_S)g$, then welfare strictly increases with g. If the actual mean-income function is $I(g) = I_S$, then welfare strictly decreases with g. Thus, given any two spending levels, there exist possible states of nature in which each yields higher welfare than the other.

This finding on the absence of dominance is important. It shows that a planner in the informational setting described here can rationalize choice of either a vanishingly small level of infrastructure spending

($g \to 0$) or a quite large level ($g \to I_S$). The former limit is optimal if public spending has no effect on mean income and the latter limit is optimal if mean income increases sufficiently rapidly with public spending. Both states of nature are possible if the planner observes a status quo policy and knows that mean income is a concave-monotone function of spending. If a planner wants to eliminate some spending levels in the range $(0, I_S)$, he must bring to bear more empirical evidence and/or stronger assumptions about the mean-income function.

4.4. Policy Choice with Various Decision Criteria

All spending levels $(0, I_S)$ being undominated, policy choice is entirely a matter of what decision criterion the planner uses.

Expected Welfare Criterion

Suppose that the planner places a subjective distribution π on the state space and maximizes expected welfare. The criterion is

$$(22) \qquad \max_{g \in (0, I_S)} \int \log[I(g) - g] d\pi.$$

The location of the maximum depends on the subjective distribution. A planner who places all probability mass on functions with slope $dI(0)/dg < 1$ knows that welfare falls as spending rises and hence will let $g \to 0$. One who places all mass on functions with slope $dI(I_S)/dg > 1$ knows that welfare rises with spending and thus will let $g \to I_S$. A planner who places positive probability on functions such that $I(g) \to 0$ as $g \to 0$ and $I(I_S) = I_S$ will associate extreme spending levels with positive probabilities of very bad welfare outcomes and hence will choose spending away from the boundary of the set $(0, I_S)$.

Maximin Criterion

The maximin criterion solves the problem

$$(23) \qquad \max_{g \in (0,\, I_S)} \; \min_{I(\cdot) \in \Gamma} \; \log[I(g) - g].$$

It was shown in (21) that the lower bound on welfare is increasing in g for $g \le g_S$ and decreasing in g for $g \ge g_S$. Hence, the maximin policy sets $g = g_S$. Thus, a maximin planner continues the status quo policy, whatever it may be.

Observe that a planner maximizing expected welfare reproduces the maximin choice if the planner places subjective probability one on the concave-monotone function $(I_S/g_S)g \cdot 1[g \le g_S] + I_S \cdot 1[g \ge g_S]$, which yields the lower bound on welfare at each value of g.

Hurwicz Criterion

The Hurwicz criterion solves the problem

$$(24) \qquad \max_{g \in (0,\, I_S)} \; \alpha \{ \max_{I(\cdot) \in \Gamma} \log[I(g) - g]\} + (1 - \alpha) \{ \min_{I(\cdot) \in \Gamma} \log[I(g) - g]\}.$$

Let H(g) denote the weighted average of maximum and minimum welfare in (24), considered as a function of g. It follows from (21) that

$$(25a) \quad g \le g_S \;\Rightarrow\; H(g) = \alpha \cdot \log(I_S - g) + (1 - \alpha)[\log(I_S - g_S) + \log(g/g_S)],$$

$$(25b) \quad g \ge g_S \;\Rightarrow\; H(g) = \alpha \cdot [\log(I_S - g_S) + \log(g/g_S)] + (1 - \alpha) \cdot \log(I_S - g).$$

Differentiation of $H(\cdot)$ by g yields

(26a) $g \le g_S \Rightarrow dH(g)/dg = -\alpha/(I_S - g) + (1-\alpha)/g = [(1-\alpha)I_S - g]/[(I_S - g)g]$,

(26b) $g \ge g_S \Rightarrow dH(g)/dg = \alpha/g - (1-\alpha)/(I_S - g) = (\alpha I_S - g)/[(I_S - g)g]$.

It follows from (26a) that, in the interval $(0, g_S]$, $H(\cdot)$ is maximized at g_S if $g_S \le (1-\alpha)I_S$ and at $(1-\alpha)I_S$ if $(1-\alpha)I_S \le g_S$. It follow from (26b) that, in the interval $[g_S, I_S)$, $H(\cdot)$ is maximized at g_S if $g_S \ge \alpha I_S$ and at αI_S if $\alpha I_S \ge g_S$.

Let g_α denote the spending level that the planner chooses with the Hurwicz criterion. The above derivation implies that

(27a) $\alpha \le g_S/I_S \le 1-\alpha \Rightarrow g_\alpha = g_S$,

(27b) $g_S/I_S \le \min(\alpha, 1-\alpha) \Rightarrow g_\alpha = \alpha \cdot I_S$,

(27c) $\max(\alpha, 1-\alpha) \le g_S/I_S \Rightarrow g_\alpha = (1-\alpha)I_S$,

(27d) $1-\alpha \le g_S/I_S \le \alpha \Rightarrow g_\alpha = \alpha I_S$ if $H(\alpha I_s) \ge H[(1-\alpha)I_S]$,

$= (1-\alpha)I_S$ if $H(\alpha I_S) \le H[(1-\alpha)I_S]$.

Thus, a planner using the Hurwicz criterion may continue the status quo policy (27a), may raise public spending to αI_S (26b and 26d) or may lower it to $(1-\alpha)I_S$ (26c and 26d).

Minimax Regret Criterion

The minimax-regret criterion is

(28) $$\min_{g \in (0, I_S)} \max_{I(\cdot) \in \Gamma} \max_{g' \in (0, I_S)} \log[I(g') - g'] - \log[I(g) - g].$$

It is not clear whether this problem has a general closed-form solution comparable to maximin and the

Hurwicz criterion. However, Section 4.5 will give a closed-form solution for a simpler version of the problem.

Although I cannot give a full characterization of minimax-regret spending, inspection of (28) shows that it must lie in the interior of the open set $(0, I_S)$. The reason is that there exist possible state of natures in which regret goes to infinity as $g \to 0$ or $g \to I_S$. The former occurs if the mean-income function is $I(g) = (I_S/g_S)\cdot g$ and the latter if it is $I(g) = I_S$. In contrast, maximum regret remains finite away from the boundaries of $(0, I_S)$.

4.5. Marginal Changes from the Status Quo

Sections 4.3 and 4.4 examined decision making in the large, supposing that the planner may choose to spend any amount in the interval $(0, I_S)$. For substantive and analytical reasons, economists often restrict attention to policies that make marginal changes from a status quo situation. Substantively, the institutional system within which a planner operates may constrain the feasible options to small deviations from the status quo. Analytically, planning problems that are complex in generality may be well-approximated by simpler problems if the feasible options are close to one another and if welfare is known to vary smoothly as a function of policy.

With these motives in mind, suppose that the planner may choose to spend any amount in an interval $[g_L, g_U]$, where $g_L \le g_S \le g_U$. Let the interval be short enough that the planner finds it plausible to assume that the mean income function $I(\cdot)$ is linear on $[g_L, g_U]$. Given that $I(\cdot)$ is increasing and that $I(g_S) = I_S$, the feasible functions on $[g_L, g_U]$ are

$$(29) \quad I(g) = \beta g + (I_S - \beta g_S), \qquad \beta \in [0, I_S/g_S].$$

Let the interval also be short enough the planner finds it plausible to approximate the logarithmic welfare function by its first-order linear expansion around g_S; that is,

$$(30)\quad \log[I(g) - g] \cong \log(I_S - g_S) + (I_S - g_S)^{-1}[dI(g_S)/dg - 1](g - g_S)$$
$$= \log(I_S - g_S) + (I_S - g_S)^{-1}(\beta - 1)(g - g_S).$$

Dropping multiplicative and additive constants that do not affect cardinal welfare, the approximate form of the planner's optimization problem is

$$(31)\quad \max_{g \in [g_L, g_U]} (\beta - 1)(g - g_S).$$

The solution to problem (31) is obvious. A planner who is willing to assume that I(·) is linear and to replace logarithmic welfare by its first-order expansion is indifferent among all feasible spending levels if $\beta = 1$. He should decrease spending to the lower bound g_L if $\beta < 1$ and increase it to the upper bound g_U if $\beta > 1$. In words, the planner should increase (decrease) spending from the status quo level if the marginal product of spending is higher (lower) than the marginal cost.

The problem of interest is choice of spending level when the planner knows only that $\beta \in [0, I_S/g_S]$. With this partial knowledge, the planner faces ambiguity. As earlier, the choice made depends on the decision criterion used.

Expected Welfare Criterion

The expected welfare criterion solves the problem

$$(32)\quad \max_{g \in [g_L, g_U]} [E_\pi(\beta) - 1](g - g_S),$$

where $E_\pi(\beta)$ is the subjective expected value of β. Thus, the planner chooses g_L if $E_\pi(\beta) < 1$ and g_U if $E_\pi(\beta) > 1$. He is indifferent among all spending levels if $E_\pi(\beta) = 1$. Observe that $E_\pi(\beta)$ acts as a certainty equivalent for β. The planner behaves as if he knew that $\beta = E_\pi(\beta)$.

Maximin Criterion

The maximin criterion solves the problem

$$(33) \qquad \max_{g \in [g_L, g_U]} \quad \min_{\beta \in [0, I_S/g_S]} (\beta - 1)(g - g_S).$$

Setting $g = g_S$ yields welfare zero for all values of β. For any $g \neq g_S$, there exist feasible values of β that make welfare negative. Hence, the maximin choice is to continue the status quo, as it was in Section 4.4.

Hurwicz Criterion

The Hurwicz criterion solves the problem

$$(34) \qquad \max_{g \in [g_L, g_U]} \alpha \left[\max_{\beta \in [0, I_S/g_S]} (\beta - 1)(g - g_S)\right] + (1 - \alpha)\left[\min_{\beta \in [0, I_S/g_S]} (\beta - 1)(g - g_S)\right].$$

Let H(g) denote the weighted average of maximum and minimum welfare in (34), considered as a function of g. It is the case that

$$(35a) \quad g \leq g_S \Rightarrow H(g) = \alpha(g_S - g) + (1 - \alpha)(I_S/g_S - 1)(g - g_S).$$

$$(35b) \quad g \geq g_S \Rightarrow H(g) = \alpha(I_S/g_S - 1)(g - g_S) + (1 - \alpha)(g_S - g).$$

Differentiation of H(·) by g yields

(36a) $g \le g_S \Rightarrow dH(g)/dg = -\alpha + (1 - \alpha)(I_S/g_S - 1) = (1 - \alpha)(I_S/g_S) - 1,$

(36b) $g \ge g_S \Rightarrow dH(g)/dg = \alpha(I_S/g_S - 1) - (1 - \alpha) = \alpha(I_S/g_S) - 1.$

It follow from (36a) that, in the interval $[g_L, g_S]$, H(·) is maximized at g_S if $g_S/I_S \le 1 - \alpha$ and at g_L if $g_S/I_S \ge 1 - \alpha$. It follow from (36b) that H(·) is maximized at g_S if $g_S/I_S \ge \alpha$ and at g_U if $g_S/I_S \le \alpha$.

Let g_α denote the spending level that the planner chooses. The above derivation implies that

(37a) $\alpha \le g_S/I_S \le 1 - \alpha \Rightarrow g_\alpha = g_S,$

(37b) $g_S/I_S \le \min(\alpha, 1 - \alpha) \Rightarrow g_\alpha = g_U,$

(37c) $\max(\alpha, 1 - \alpha) \le g_S/I_S \Rightarrow g_\alpha = g_L,$

(37d) $1 - \alpha \le g_S/I_S \le \alpha \Rightarrow g_\alpha = g_U$ if $H(g_U) \ge H(g_L),$

$= g_L$ if $H(g_U) \le H(g_L).$

Thus, the planner may continue the status quo policy (37a), may raise public spending to g_U (36b and 36d) or may lower it to g_L (36c and 36d).

Finding (37a) is identical to (27a) in Section 4.4. Findings (37b)-(37d) have the same premises as (27b)-(27d) and have the same conclusions regarding raising or lower spending. However, the two sets of findings differ in the specifics of the choices made.

Minimax-Regret Criterion

The minimax-regret criterion is

(38) $$\min_{g \in [g_L, g_U]} \max_{\beta \in [0, I_S/g_S]} \max_{g' \in [g_L, g_U]} (\beta - 1)(g' - g_S) - (\beta - 1)(g - g_S).$$

For each $g \in [g_L, g_U]$ and $\beta \in [0, I_S/g_S]$, regret is

(39) $$\max_{g' \in [g_L, g_U]} (\beta - 1)(g' - g_S) - (\beta - 1)(g - g_S) = (\beta - 1) \max_{g' \in [g_L, g_U]} (g' - g)$$

$$= (\beta - 1)(g_L - g)\cdot 1[\beta < 1] + (\beta - 1)(g_U - g)\cdot 1[\beta > 1].$$

Hence, criterion (38) reduces to

(40) $$\min_{g \in [g_L, g_U]} \max_{\beta \in [0, I_S/g_S]} (\beta - 1)(g_L - g)\cdot 1[\beta < 1] + (\beta - 1)(g_U - g)\cdot 1[\beta > 1].$$

For each $g \in [g_L, g_U]$, maximum regret of g across $\beta \in [0, 1]$ occurs when $\beta = 0$ and equals $(g - g_L)$. Maximum regret across $\beta \in [1, I_S/g_S]$ occurs when $\beta = I_S/g_S$ and equals $(I_S/g_S - 1)(g_U - g)$. Thus, maximum regret of g across all feasible β is $\max[(g - g_L), (I_S/g_S - 1)(g_U - g)]$. Hence, criterion (40) reduces to

(41) $$\min_{g \in [g_L, g_U]} \max[(g - g_L), (I_S/g_S - 1)(g_U - g)].$$

Finally observe that as g increases from g_L to g_U, the expression $(g - g_L)$ increases linearly from 0 to $(g_U - g_L)$ and the expression $(I_S/g_S - 1)(g_U - g)$ decreases linearly from $(I_S/g_S - 1)(g_U - g_L)$ to 0. It follows that the minimax-regret choice for g solves the equation

(42) $$g - g_L = (I_S/g_S - 1)(g_U - g).$$

The solution is

(43) $g = (g_S/I_S)g_L + (1 - g_S/I_S)g_U$.

Thus, the location of the minimax-regret choice within the interval $[g_L, g_U]$ varies inversely with the status-quo ratio of public spending to mean income.

Numerical Illustration

A numerical illustration helps to see how the chosen spending level depends on the decision criterion used. Suppose that status quo spending per capita on infrastructure is g_S = \$8,000 and mean income is I_S = \$40,000. Thus, the status quo tax rate is $g_S/I_S = 0.2$ and the upper bound on β is $I_S/g_S = 5$. Let the feasible range of spending levels be $[g_L, g_U]$ = [\$6,000, \$10,000].

In this setting, a planner who places a uniform distribution on β and maximizes expected welfare acts as if β = 2.5 and, therefore, chooses to spend \$10,000. A planner using the maximin criteria continues to spend the status quo \$8,000. One using the Hurwicz criterion with α = 0.5 satisfies the premise of (37b) and, hence, chooses to spend \$10,000. A planner using the minimax-regret criterion chooses to spend \$9,200.

5. Methodological and Substantive Conclusions

This paper yields methodological and substantive conclusions. I explain below.

5.1. Methodology for Normative Study of Size of Government

When performing normative research on size of government, economists need not study optimization problems whose solution requires far more knowledge than is credible to assert. Decision theory provides a suitable formal framework for study of planning under ambiguity. Section 3 showed how to apply this framework in principle. Section 4 showed that it yields simple findings in an illustrative setting.

A large unresolved issue is how to evaluate policies that may not yield balanced budgets. To bypass the complexity of dynamic policy evaluation under ambiguity, I focused on settings where a planner can ensure budget balance by choosing policy components sequentially. However, these settings have special features that do not match a reality in which nations regularly experience budget surpluses and deficits.

Another direction for methodological work is to perform further analyses in the style of Section 4, ones that yield simple and informative findings. I think that progress is possible, but it will require juxtaposition of policy options and state spaces that yield tractable scenarios. Researchers studying taxation and public spending have found it difficult to characterize general solutions to optimal planning problems, so they have regularly studied special cases. Analysis of special cases can also contribute to study of planning under ambiguity, which typically is more complex than traditional optimization. Numerical analysis of problems that defy simple analysis may also be helpful.

5.2. The Difficulty of Drawing Substantive Conclusions about Size of Government

The analysis of Section 4 demonstrated that a planner with considerable knowledge of population income-leisure preferences and partial knowledge of the productivity of infrastructure spending can reasonably choose a wide range of spending levels. The choice made depended on the decision criterion used to cope with ambiguity. This is inevitable given that no spending level was dominated.

The informational setting of Section 4 is far more benign than the one that societies face in actuality. We have relatively little knowledge of population preferences and the productivity of spending. Hence, it is easy to rationalize a small or large government. Moreover, societies face the conceptually and technically difficult problem of dynamic policy choice under ambiguity, which I circumvent entirely in this paper.

Consider first our knowledge of income-leisure preferences, holding public spending fixed. Recall that standard economic theory, which assumes only that persons prefer more income and more leisure, does not predict the response of time allocation to income taxation. To the contrary, it shows that a worker may rationally respond in disparate ways. As tax rates increase, a person may rationally decide to work less, work more, or not change his time allocation at all.

To learn about preferences, empirical research has combined standard theory with strong preference assumptions and with observation of time-allocation decisions under a status quo policy. In Manski (2012), I showed that revealed-preference analysis using only the basic assumption that income and leisure are both desirable may bound a person's time allocation under a proposed policy or may have no implications, depending on the tax schedules and the person's status quo time allocation. I next explored the identifying power of two classes of preference assumptions. One assumed that groups of persons who face different choice sets have the same preference distribution. The second restricted the shape of this distribution. The generic finding was partial identification of preferences. Tight inferences become feasible only when one imposes strong preference assumptions that are difficult to motivate.

Next consider preferences for public spending on amenities. It is easy to show that basic revealed preference analysis yields no predictions of time allocation when a proposed policy changes status quo spending. Given this, researchers have found it is analytically convenient to assume that preferences are separable in spending for amenities, as I did in Section 3. However, I am aware of no evidence for the realism of this assumption. Over thirty years ago, Wildasin (1979) observed (p. 63-64): “The proper way of taking the effects of distortionary taxes into account in evaluating public expenditure depends sensitively on complement-substitute relations between public and private goods.” He went on (p. 64): “Most bothersome of all is the fact that we have very little empirical information on the interaction between public good provision and private demand.” To my knowledge, this observation remains accurate today.

Finally, consider the productivity of infrastructure spending. The planner of Section 4 assumed that wage is a person-specific constant multiplied by a concave-monotone function of such spending. This multiplicative structure was purely an analytical convenience. I am aware of no evidence that infrastructure spending affects wages in this distribution-neutral manner. The assumption that wages increase with infrastructure spending but with diminishing returns has some plausibility, but it is easy to conjecture reasons why it may not reflect reality. For example, fixed costs of production of infrastructure may generate increasing returns over some range of spending levels. Or so-called “regulatory burden” may generate negative returns at sufficiently high spending levels.

Empirical macroeconomists have sought to shed light on the productivity of public spending by studying time-series and cross-country variation in aggregate output and public spending. See Aschauer (1989) for an influential contribution and Gramlich (1994) for a review and critique of the literature. Unfortunately, this body of research uses highly restrictive model specifications imposed to enable application of traditional econometric methods for analysis of linear models. As with empirical research on labor supply, the maintained assumptions are difficult to motivate.

To sum up, I think that present-day economics does not provide the knowledge required to draw credible conclusions about the desirable size of government. The only way to make progress is to improve current knowledge of population preferences and the productivity of public spending. I worry that conventional academic research, in which independent investigators make decentralized decisions about subjects of study and write articles for publication as stand-alone journal articles, will not yield the cumulative body of knowledge that societies need to make informed decisions. Initiation of research programs by institutions with a suitably long-run perspective—including central banks, government ministries, and private foundations—may help to make progress possible. Governments could enrich the available empirical evidence by experimentally varying their spending and taxation policies across space or time.

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Figure 1: Feasible Mean-Income Functions