



Labor-Market Fluctuations and On-The-Job Search

Éva Nagypál

Faculty Fellow, Institute for Policy Research
Assistant Professor of Economics

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Abstract

This paper argues that a model of the aggregate labor market that incorporates the observed extent of job-to-job transitions can explain all the cyclical volatility in vacancy and unemployment rates in U.S. data in response to shocks of the observed magnitude. The key to this result is the complementarity between on-the-job search and costly hiring that leads employers to expect a higher payoff from recruiting employed searchers. This higher expected payoff explains why firms recruit more when the number of employed searchers is high during periods of low unemployment.

1. INTRODUCTION

The ongoing coexistence of unemployed workers and vacant positions has lent support in macroeconomics to a frictional view of labor markets wherein the matching of searching workers and vacant positions takes time. According to this view, unemployment is low during boom periods because there are many firms recruiting and it takes little time for workers to become employed. Recent research has suggested, however, that, in the context of a simple matching model, the observed variation in the extent of recruitment cannot be understood as a response to changes in the productivity and duration of prospective employment relationships (Shimer (2005), Mortensen and Nagypál (2007b)). A critical question remains, namely, why do firms recruit so much more in a boom?

This paper offers an answer to this question by arguing that workers search not only when unemployed, but also while they are on a job. Due to the possibility of being able to stay with their current employer, employed searchers are more selective in accepting job offers than are unemployed workers. They will only accept those jobs that are particularly attractive and that they are, therefore, unlikely to quit later on. This unique aspect makes the employment relationships formed by employed searchers last longer than those formed by unemployed searchers and allows a recruiting firm to recoup the cost of hiring over a longer period of time, thereby making it more profitable for firms to recruit employed searchers.

In a matching model with on-the-job search, the higher profitability from recruiting employed searchers leads to a multiplier effect. When firms recruit more, it is a good time for employed workers to try and improve their lot by searching for a better job. In turn, when the pool of searchers consists mostly of employed workers, it is a good time for firms to recruit. This interplay between recruitment and on-the-job search is empirically very important: the fraction of new jobs that are filled by workers who were employed immediately prior to starting their new jobs is around one half and very strongly procyclical (Nagypál (2005b)). When I calibrate my model to match this fact, the model can explain *all* the observed volatility in the vacancy and unemployment rates, given a hiring cost of just over a week's of wages.¹

¹These results significantly improve upon those of Mortensen and Nagypál (2007b) whose calibrated matching model with the same features but *without* on-the-job search can explain only 52% of the volatility of the vacancy rate, a key endogenous variable in matching models.

It is important to emphasize that neither a hiring cost nor on-the-job search by itself could generate the same amount of volatility. Rather, it is the interaction of the two that gives rise to my results. Without including on-the-job search, not only does the model fail to account for the large number of jobs filled by previously employed searchers, it also explains only 62% of the observed volatility in the vacancy rate. Without a hiring cost, the presence of on-the-job search actually decreases amplification so that the model can only explain 37% of the volatility of the vacancy rate. In this case, it is more profitable for firms to recruit unemployed workers and the multiplier effect discussed above is absent.

Section 2 introduces my model of the labor market with on-the-job search and costly hiring. Variation in the idiosyncratic job-satisfaction provided by different job matches leads to a desire by workers to seek out more attractive and thereby longer lasting jobs and undertake job-to-job transitions. In Section 3, I calibrate the model parameters to match the observed turnover in the labor market, including the magnitude of job-to-job transitions. In Section 4, I highlight the key mechanisms driving my results by characterizing a simplified version of my model. I show analytically that a higher profitability from recruiting employed searchers naturally arises in the presence of costly hiring: the longer expected duration of matches formed with employed workers makes the effective cost of hiring such workers smaller. I also show that costly hiring and on-the-job search interact in amplifying the effect of aggregate shocks on key labor-market variables. Section 5 provides empirical support for the notion that the expected duration of employment relationships formed by employed workers is longer and discusses the role of job-destruction shocks. Section 6 relates my findings to the existing literature.

2. MODEL

The model I consider is a generalization of the textbook matching model (Pissarides (2000)) extended to include a hiring cost and on-the-job search. There is a large measure of ex-ante identical firms, whose objective is to maximize their expected discounted profits using the discount rate r . Firms are free to enter the market to create employment matches by posting a vacancy at flow cost $c > 0$ in order to recruit a worker. If a firm succeeds in recruiting a worker, it has to pay

hiring cost $H > 0$ to employ the worker.² Subsequently, the flow output of an employment match is given by labor productivity $p > 0$ until the match is either destroyed for exogenous reasons at rate $\delta > 0$ or the worker quits to take another job as a consequence of on-the-job search.

There is a unit measure of ex-ante identical infinitely-lived workers, whose objective is to maximize their expected discounted payoff using the same discount rate r . Let $b < p$ then be the utility flow that a worker receives while unemployed, derived from leisure and from unemployment-insurance benefits. Suppose that each job match, in addition to paying a wage, provides an idiosyncratic job-satisfaction value to the worker that is determined by such non-wage characteristics as the pleasure from working on the tasks prescribed, the appeal of co-workers, and the convenience of the job's location and schedule. Specifically, let the utility flow of an employed worker equal $w + \mu$ where w is the wage and μ is an i.i.d. random variable representing the idiosyncratic taste component characterized by the continuously differentiable c.d.f. $F : [\underline{\mu}, \bar{\mu}] \rightarrow [0, 1]$ and survival function $\bar{F} = 1 - F$. Assume that the idiosyncratic component is only observable by the worker when a worker and a firm meet.³

There is a single matching market with a matching function that determines the number of meetings between workers and firms as a function of the total search effort of workers, \bar{s} , and the number of vacancies posted, v . The matching function $m(\bar{s}, v)$ has constant returns to scale, is strictly increasing, and is continuously differentiable in both of its arguments, and has a constant elasticity with respect to vacancies, denoted by ν . The matching rate of workers *per unit of search effort* can be written as

$$\lambda = \frac{m(\bar{s}, v)}{\bar{s}} = m\left(1, \frac{v}{\bar{s}}\right) = m(1, \theta),$$

where $\theta = \frac{v}{\bar{s}}$ is market tightness in the model. The matching rate for firms is then $\eta = \lambda/\theta$. Both unemployed and employed workers choose their search effort s given the search cost function $k(s) = \kappa s^{1+\rho}$, where $\kappa > 0$ and $\rho > 0$. A worker exerting search effort s contacts vacancies at rate λs . If a worker and a firm are matched, they have to decide whether to form the relationship.

²In the presence of on-the-job search and rejections of some potential employment relationships by workers, there is a qualitative difference between costs that firms need to pay to generate a contact with a worker and costs that firms need to pay only when a worker is actually hired.

³The assumption that is important for my results is that the firm does not observe the idiosyncratic component prior to expending the hiring cost.

Unlike in the textbook model, not all meetings result in the formation of an employment relationship due to the presence of heterogeneity in match values and on-the-job search.

I assume that wages are set by continuous Nash bargaining over the division of the surplus without the possibility to commit to the future sequence of wages.⁴ In the spirit of Hall and Milgrom (2007), I assume that the disagreement payoff of the worker and the firm is delay. I also assume that the worker enjoys the idiosyncratic payoff μ as long as the relationship continues, irrespective of whether an agreement over the wage is reached or not at a particular point in time. As I show in the Appendix, the outcome of this bargaining game is simply

$$(1) \quad w_t = b + \beta(p - b),$$

where β denotes the worker's bargaining share. The wage is thus independent of match quality since, given the assumed bargaining protocol, the firm cannot extract any of the rents that the worker enjoys from having a good match. This result means that job-to-job transitions in the model are driven by heterogeneity among matches in their idiosyncratic payoff to the worker.⁵ The assumed bargaining protocol also ensures that an employed worker who searches while on a job cannot extract all the rents from the less-appealing job when that worker is facing a choice between two jobs.

2.1. Characterization of steady state. In what follows, I focus on the steady state of this model. I maintain that the parameters of the model are such that $p > w > b - \bar{\mu}$, so that some employment relationships are formed. The continuous-time Bellman equation that characterizes the value of being employed with idiosyncratic value μ is

$$(2) \quad rW(\mu) = \max_{s \geq 0} \left\{ w + \mu - k(s) + \lambda s \int_{\mu}^{\bar{\mu}} \max[W(\mu') - W(\mu), 0] dF(\mu') + \delta(U - W(\mu)) \right\},$$

where U is the value of unemployment. The flow utility from working is $w + \mu$. If the worker encounters a new firm, which happens at rate λs , she needs to decide whether to form the new

⁴Given the lack of commitment to the future sequence of wages, which determines the incentive to search on-the-job, the non-convexity of the Pareto frontier discussed in Shimer (2006) does not arise in this setting.

⁵If the idiosyncratic component remains unobservable to the firm even after paying the hiring cost, then one can adopt the argument of Menzio (2005a) (building on the work of Grossman and Perry (1986) and Gul and Sonnenschein (1988)) to show that the outcome of an asymmetric-information alternating-offers bargaining game where the parties bargain continuously over the division of the net match product is immediate trade at terms that are independent of the informed party's type in the limit as the time between offers becomes zero.

match, given its idiosyncratic component μ' drawn from the distribution F . Moreover, the worker suffers a loss of asset value due to exogenous job-destruction at rate δ . Equation (2) defines a contraction; therefore, the Contraction Mapping Theorem implies that, given the assumptions on $F(\cdot)$ and $k(\cdot)$, $W(\cdot)$ is strictly increasing and is continuously differentiable. This feature, in turn, implies that acceptance decisions have the reservation property, with the idiosyncratic value of the current match being the reservation value of an employed worker and μ_r being the reservation value of an unemployed worker. Differentiating with respect to μ on both sides of the worker's asset equation, using the envelope theorem, and rearranging gives

$$(3) \quad W'(\mu) = \frac{1}{r + \delta + \lambda s(\mu) \bar{F}(\mu)}.$$

Because the opportunities to search are the same regardless of employment status, there is no option value of search lost or gained when a worker accepts a job. The reservation value of the idiosyncratic component μ is, therefore, simply the value that compensates the worker for any forgone income:

$$\mu_r = \begin{cases} b - w & \text{if } b - w > \underline{\mu} \\ \underline{\mu} & \text{if } b - w \leq \underline{\mu} \end{cases}$$

The continuous-time Bellman equation characterizing the value of being unemployed is

$$rU = \max_{s \geq 0} \left\{ b - k(s) + \lambda s \int_{\mu_r}^{\bar{\mu}} (W(\mu') - U) dF(\mu') \right\}.$$

Denote the search effort of unemployed workers by s_u and the search effort of employed workers as a function of their idiosyncratic component by $s(\mu)$. The first-order conditions characterizing workers' search effort choices are given by

$$k'(s_u) = \lambda \int_{\mu_r}^{\bar{\mu}} (W(\mu') - U) dF(\mu')$$

and

$$(4) \quad k'(s(\mu)) = \lambda \int_{\mu}^{\bar{\mu}} (W(\mu') - W(\mu)) dF(\mu') = \lambda \int_{\mu}^{\bar{\mu}} W'(\mu') \bar{F}(\mu') d\mu',$$

where the last equality follows from integration by parts. Using the particular functional form for $k(\cdot)$ and Equation (3), and differentiating both sides of Equation (4) with respect to μ results in

the differential equation

$$(5) \quad s'(\mu) = -\frac{\lambda \bar{F}(\mu) s(\mu)^{1-\rho}}{(1+\rho)\rho\kappa(r+\delta+\lambda s(\mu)\bar{F}(\mu))}.$$

This differential equation together with the boundary condition $\lim_{\mu \rightarrow \bar{\mu}} s(\mu) = 0$ has a unique solution which fully characterizes the search decision of employed workers. Clearly, $s_u = s(b-w)$ and $s(\cdot)$ is a strictly decreasing function so that workers with a higher value of μ search less intensively.

Given that an employed worker quits a match with an idiosyncratic component μ at rate $\lambda s(\mu)\bar{F}(\mu)$ and that free entry drives the value of a vacant job to zero in equilibrium, the value of a firm with a filled job with idiosyncratic component μ solves

$$rJ(\mu) = p - w - (\delta + \lambda s(\mu)\bar{F}(\mu)) J(\mu).$$

Given the wage in Equation (1),

$$J(\mu) = \frac{(1-\beta)(p-b)}{r+\delta+\lambda s(\mu)\bar{F}(\mu)}.$$

Notice that the employer's "total discount rate" with which it discounts future profit flows includes the quit rate. Since the quit rate is decreasing with μ , the value of a match to the firm increases in μ . So while firms do not get any direct benefit from the non-wage component, they expect to make more profits on matches with workers who have a high value of job satisfaction μ . All of this effect is coming through the effect of job satisfaction on the quit rate, by lowering both a worker's incentive to search and their probability of accepting an outside offer.⁶

Free entry equalizes the cost and benefit of vacancy posting, so that

$$(6) \quad c = \eta(\gamma\Pi_e + (1-\gamma)\Pi_u),$$

where Π_e and Π_u are the expected payoff from contacting an employed and unemployed worker, respectively, and γ is the probability that a contacted searcher is employed. The expected payoffs Π_e and Π_u are different due to the different acceptance behavior of employed and unemployed

⁶If the worker did not enjoy the idiosyncratic component during delay, the bargaining protocol assumed would allow the firm to extract some of the payoff from having a high μ match, thus adding an additional channel through which μ would increase the firm's payoff.

searchers: while unemployed searchers accept all new matches with an idiosyncratic component above μ_r , employed searchers are more selective. Hence,

$$(7) \quad \Pi_e = \int_{\mu_r}^{\bar{\mu}} (J(\mu) - H) A_e(\mu) dF(\mu)$$

$$(8) \quad \Pi_u = \int_{\mu_r}^{\bar{\mu}} (J(\mu) - H) dF(\mu),$$

where $A_e(\mu)$ is the probability that an employed searcher accepts a match with idiosyncratic component μ .

As in any model with on-the-job search, the distribution of employed workers over job characteristics differs from the distribution over vacant jobs as a consequence of selection. Specifically, because employed workers only move to jobs with higher values of μ , and workers only accept jobs above the reservation value μ_r , the measure of workers employed in jobs with an idiosyncratic component less than or equal to μ , denoted by $G(\mu)$, and the measure of unemployment, denoted by u , satisfy the following steady-state balance equations that arise from equating flows into and out of the relevant pool of workers:

$$(9) \quad \lambda s_u (F(\mu) - F(\mu_r)) u = \delta G(\mu) + \lambda \bar{F}(\mu) \int_{\mu_r}^{\mu} s(\mu') dG(\mu')$$

and

$$\delta(1 - u) = \lambda s_u \bar{F}(\mu_r) u.$$

The steady state unemployment rate is then

$$u = \frac{\delta}{\delta + \lambda s_u \bar{F}(\mu_r)},$$

while differentiating both sides of Equation (9) with respect to μ and rearranging gives

$$(10) \quad G'(\mu) = \frac{u \lambda s_u - \delta G(\mu)}{\delta + \lambda s(\mu) \bar{F}(\mu)} \frac{F'(\mu)}{\bar{F}(\mu)}.$$

This differential equation, together with the boundary condition $G(\mu_r) = 0$, has a unique solution that fully characterizes the distribution of workers. Given the distribution G , the probability that

a match of quality μ is accepted by an employed searcher can be expressed as

$$A_e(\mu) = \frac{\int_{\mu_r}^{\mu} s(\mu') dG(\mu')}{\int_{\mu_r}^{\bar{\mu}} s(\mu') dG(\mu')},$$

while the probability that a worker that a vacant job encounters is employed is

$$\gamma = \frac{\int_{\mu_r}^{\bar{\mu}} s(\mu') dG(\mu')}{u s_u + \int_{\mu_r}^{\bar{\mu}} s(\mu') dG(\mu')}.$$

3. QUANTITATIVE RESULTS

In this section, I assess the business-cycle performance of the above model by using a log-linear approximation around the non-stochastic steady-state of the equilibrium conditions. In the Appendix, I discuss the extent to which this exercise gives a good approximation to the response of the model to aggregate disturbances in its full dynamic stochastic version. In particular, I use a log-linear approximation around the steady state for arbitrary endogenous variables x and y

$$\begin{aligned} \Delta \ln x &= \alpha_{px} \Delta \ln p + \alpha_{\delta x} \Delta \ln \delta \\ \Delta \ln y &= \alpha_{py} \Delta \ln p + \alpha_{\delta y} \Delta \ln \delta, \end{aligned}$$

where the α coefficients are functions of numerically calculated derivatives. I then approximate the model-implied volatility of $\ln x$ by

$$\sigma_x^2 = E [(\Delta \ln x)^2] = \alpha_{px}^2 \sigma_p^2 + 2\alpha_{px} \alpha_{\delta x} \rho_{\delta p} \sigma_\delta \sigma_p + \alpha_{\delta x}^2 \sigma_\delta^2$$

and the correlation of $\ln x$ and $\ln y$ by

$$\rho_{xy} = \frac{cov(x, y)}{\sigma_x \sigma_y} = \frac{\alpha_{px} \alpha_{py} \sigma_p^2 + (\alpha_{px} \alpha_{\delta y} + \alpha_{\delta x} \alpha_{py}) \rho_{\delta p} \sigma_\delta \sigma_p + \alpha_{\delta x} \alpha_{\delta y} \sigma_\delta^2}{\sigma_x \sigma_y}.$$

I focus on two sources of business-cycle volatility: changes in labor productivity, p , and changes in the job-destruction rate, δ .⁷ Shimer (2005) has argued that changes in the job-destruction

⁷I use the term job-destruction rate and separation rate interchangeably in this paper because the notion of a job-destruction shock is more expressive and because in the one worker-one firm setup of a matching model there is no difference between the job-destruction rate and the rate at which employed workers separate from their job into unemployment. In terms of the data, what Shimer (2005) reports is the separation rate.

rate cannot be sources of aggregate fluctuations in a matching model since such changes induce a positive correlation between the unemployment and the vacancy rates, while the data show a strong negative correlation. Without on-the-job search, the pool of searchers is made up exclusively of unemployed workers. As the number of unemployed workers rises in response to an increase in the job-destruction rate, it becomes relatively easy for a vacancy to be filled. Thus the number of vacancies rises despite the fall in the vacancy-unemployment ratio. This counterfactual implication need not present in a model with on-the-job search where the pool of searchers varies less than the pool of unemployed workers. In particular, notice that in the extreme case where all employed workers search with the same effort as unemployed workers, a change in the unemployment rate does not have any effect on the total amount of search effort, which is simply proportional to the labor force. Changes in the job-destruction rate thus become a potential source of aggregate fluctuations in the presence of on-the-job search. This possibility is not only a theoretical one, but also a quantitatively important one in light of the significant cyclical variation in the rate of separations into unemployment observed in the data.⁸

3.1. Benchmark calibration. For my benchmark calibration, I set the discount rate to $r = 0.012$, so that the unit of time in the model is a quarter. I normalize $p = 1$, and use the values reported by Shimer (2005) to calibrate $\delta = 0.10$, $\lambda = 1.355$, $\sigma_p = 0.02$, $\sigma_\delta = 0.075$, and $\rho_{p\delta} = -0.524$. As is well known (see the discussion in Mortensen and Nagypál (2007b)), the flow payoff during unemployment is a controversial variable in the calibration of matching models. The most careful estimate is due to Hall and Milgrom (2007). They use utility parameter values based on the empirical literature on household consumption and labor supply and reports of the effective replacement ratio to estimate the value of b to be 0.71. This is the number I use in my benchmark calibration, but I also report below results for $b = 0.40$, the value used by Shimer (2005).

I set the parameters that relate to search effort as follows. I set the curvature of the search-cost function to match the observation in Nagypál (2005b) that, in the aggregate, the magnitude of the quit rate is as large as the rate at which workers transit from employment to unemployment. This requires setting $\rho = 1.35$. I examine the sensitivity of my results to the choice of this parameter below. For the distribution of idiosyncratic values, I use a uniform distribution on $[-\sigma, \sigma]$ and set

⁸The total separation rate, i.e., the rate at which workers separate from their employers, does not vary much due to the procyclicality of the quit rate.

$\sigma = 0.1$.⁹ There is no good empirical counterpart to guide the choice of σ , so I perform sensitivity analysis with respect to its value below.

I set the worker’s share of the net match product to 90% to get the level of wages to be similar to the one in the standard model. This parameter has no effect on the model’s amplification properties through profits, since $\frac{d \ln(p-w)}{d \ln p} = \frac{p}{p-b}$ and $\frac{\partial \ln(p-w)}{\partial \ln \lambda} = 0$, independent of β . It does influence the level of wages, however, which, in turn, affects the response of unemployed workers’ search effort to changes in parameters. To determine the strength of this channel, I examine the sensitivity of my results to the choice of β below.

Although there is a consensus in the literature that hiring costs are important, there is no authoritative estimate of their magnitude. Therefore, I set the hiring cost to match the volatility of vacancies in my benchmark calibration. This requires setting the hiring cost to 2.9 quarter’s of flow profits and implies that, in order to recoup the initial cost of employing a worker, a firm needs to continue employment for at least three quarters. Given the calibrated wage, this hiring cost is equal to 1.13 weeks’ of wages. Given this hiring cost, the payoff from contacting an employed worker (Π_e) is 67% higher than the payoff from contacting an unemployed worker (Π_u). In light of the important role that the hiring cost plays in the model, I also report results for different hiring costs below.

Another important variable in the calibration is the elasticity of the matching function with respect to vacancies (see Mortensen and Nagypál (2007b)). Shimer (2005) calibrates it to $\nu = 0.28$ by calculating the elasticity of the job-finding rate with respect to the vacancy-unemployment ratio. With on-the-job search, market tightness is no longer equal to the vacancy-unemployment ratio, so this value is not the appropriate one to use. In the extreme case when employed workers contact vacancies at the same rate as unemployed workers, market tightness is proportional to vacancies. Given Shimer’s data, the elasticity of the job-finding rate with respect to vacancies is

$$\nu = \frac{\rho_{\lambda v} \sigma_{\lambda}}{\sigma_v} = \frac{0.897 \times 0.118}{0.202} = 0.52.$$

⁹It is worth noting that the choice of the distribution function — from the class of generally used distribution functions — does not have a large impact on the amplification properties of the model. Use of a truncated normal distribution, for example, gives similar results.

The case of endogenous search effort where employed workers search less than unemployed workers is in between these two extremes, so I set $\nu = 0.40$. This value is also at the midpoint of the empirically plausible values reported by Petrongolo and Pissarides (2001).

Finally, I choose the remaining variables to generate a job-finding rate of 1.355. In particular, I set the parameter κ so that the search effort of unemployed workers, s_u , is unity¹⁰ and set the contact rate per unit of search effort, λ , equal to the job-finding rate $f = \lambda s_u$. Once the value of κ is determined, I choose the cost of posting a vacancy and the scale parameter of the Cobb-Douglas matching function to make sure that the free-entry condition holds for the calibrated values of λ and H and the implied vacancy-unemployment ratio is 0.5, the empirical value reported by Faberman (2005). Note, however, that these last two parameters do not appear in the log-linearized system and hence do not affect the volatilities implied by the model.

The equilibrium values of interest using the benchmark calibration are reported in Table 1. The unemployment rate, the vacancy rate, and the job-finding rate of unemployed workers are of course exactly equal to their calibrated values. The calibrated quit rate is 0.101. Due to job-to-job transitions, the steady-state distribution of idiosyncratic values first-order stochastically dominates the distribution of the initial draw of idiosyncratic values, with the average idiosyncratic component equal to 5.5% of output. Even though there are many more employed searchers than there are unemployed searchers, a firm has a 29.2% chance of contacting an unemployed searcher, due to the higher search effort of unemployed workers. Due to the lower acceptance rate of employed searchers, they account for only 50.1% of new hires, even though they represent 70.8% of all contacts.

unemployment rate	6.87%
vacancy rate	3.44%
job-finding rate	1.355
quit rate	0.101
average idiosyncratic component	0.055
prob. of contacting employed searcher	70.8%
fraction of new hires previously employed	50.1%

TABLE 1. Equilibrium value of relevant labor-market variables using the benchmark calibration.

¹⁰Such a value of κ always exists and gives a convenient normalization, since κ scales the equilibrium value of the contact rate, λ , and of the search effort function, $s(\cdot)$.

In Figure 4, I plot the search effort chosen by workers with different idiosyncratic values, the density of the distribution of initial idiosyncratic component draws, F , and of the endogenous equilibrium distribution of employed workers across idiosyncratic components, G . Due to the selection towards higher idiosyncratic values through job-to-job transitions, the second distribution first-order stochastically dominates the first.

3.2. Business-cycle volatility. The first column of Table 2 reports the volatility and correlation of the labor-market variables of interest implied by the model in the benchmark calibration. To facilitate the comparison, in the last column, I report the observed volatility of the labor-market variables reported by Shimer (2005).

	Benchmark model	Short run	Without hiring cost	No OTJ search	U.S. data
Hiring cost	$\bar{H} = 2.9$	$\bar{H} = 2.9$	$\bar{H} = 0$	$\bar{H} = 2.9$	Shimer (2005)
On-the-job search	yes	yes	yes	no	
contact rate, λ	0.0991	0.0998	0.0393	0.0683	—
job-finding rate, f	0.1376	0.1374	0.0728	0.0947	0.1180
unemp. rate, u	0.1870	0.1887	0.1266	0.1504	0.1900
vacancy rate, v	0.2028	0.2619	0.0739	0.1265	0.2020
quit rate, q	0.0359	0.1489	0.0237	—	—
corr(u,v)	-0.962	-0.985	-0.820	-0.900	-0.894
corr(u,q)	-0.686	-0.994	0.442	—	—

TABLE 2. Volatility and correlation of relevant labor-market variables in the benchmark model, in the short run, without a hiring cost, without on-the-job search, and using U.S. data.

The results in the first column show that the job-finding rate responds more to variation in p and δ than to the contact rate: the optimal search effort of unemployed workers increases in good times in response to the increase in the wage and the contact rate. Taking the search effort channel into account means that the benchmark model predicts slightly more variation in the job-finding rate than what is observed in the data.¹¹ The presence of the observed amount of variation in the job-destruction rate implies that the model can explain all the observed variation in the unemployment rate. The benchmark model also explains the observed variation in the vacancy rate. While this is true by construction, it is important to note that to get this result, I did not need to resort to an implausibly high value of leisure nor to a high hiring cost. Moreover, the ability to explain the

¹¹Merz (1995) also finds that incorporating a search effort channel increases the volatility of the unemployment rate.

variability of the vacancy rate turns out to be unique to the benchmark model that features *both* on-the-job search and a hiring cost. As for the correlation of unemployment and vacancies, it is somewhat stronger than in the data. This strong negative correlation contrasts with the findings of Shimer (2005). The full dynamic stochastic version of the model and lags in the adjustment of vacancies as in Fujita and Ramey (2007b) could undo some of the almost perfect negative correlation predicted by the model.

As for the quit rate, it shows relatively little volatility and a weaker negative correlation with the unemployment rate. (The probability that a firm encounters an employed searcher is strongly procyclical though, as employment is procyclical.) This result is due to two countervailing effects in the model in response to an increase in the contact rate. First, an increase in the contact rate increases the rate at which employed workers meet potential new employers, both directly and indirectly by encouraging more search effort, and thereby increasing the quit rate. Second, an increase in the contact rate shifts the distribution of employed workers in the steady state toward higher idiosyncratic values where workers are less likely to find a better offer, thereby decreasing the quit rate. This upward shift in the distribution is further enhanced by the decrease in the job-destruction rate that takes place during good times. While this second effect is present in the steady state, it takes a relatively long time to unfold, given that employment spells on average last for ten quarters in the model.

To assess the short-run response of the model then, in the second column, I report results for the same model when the distribution across idiosyncratic values of employed workers is left unchanged. Thus, in the short-run, only the composition of the searchers between unemployed and employed changes. The largest increase is in the response of the vacancy and quit rates.¹² Due to the lack of an upward shift in the distribution of idiosyncratic values, there are more employed searchers with a higher acceptance rate in the short run than in the long run, which increases the quit rate and also the number of vacancies for a given market tightness.

In the third column of Table 2, I report the same statistics once the hiring cost is removed. The predicted variability in the job-finding, unemployment, and vacancy rates declines substantially, to 62%, 67%, and 36% of their observed values, respectively. The variability of the quit rate also

¹²Also, while the total rate of separation from employment and the unemployment rate covary positively in the long run, they covary negatively in the short run. In both cases, the volatility of the total separation rate is small, 0.0364 in the long run and 0.0462 in the short run.

declines slightly, and its correlation with unemployment becomes counter-factually positive. These results show that the presence of the hiring cost is crucial in generating the results of the benchmark model.

Finally, to examine how much of the response of the benchmark calibration is due to the presence of the complementarity between on-the-job search and the hiring cost, in the fourth column of Table 2, I examine the model with a search effort margin, but without on-the-job search.¹³ Both the volatility of the contact rate and the job-finding rate is about 31% lower than in the benchmark model, in turn implying a somewhat lower volatility for the unemployment rate, primarily due to the lower estimate of ν without on-the-job search. The volatility of the vacancy rate, however, is 38% lower in the model without on-the-job search, a result that does not hinge on the estimate of ν (with $\nu = 0.40$, the volatility would still only be 0.1299). Therefore, taking account of on-the-job search and the complementarity between on-the-job search and the hiring cost in generating labor-market volatility is important for two reasons. First, it allows for accounting for the observed amount of labor-market volatility within an empirically grounded framework that takes into account the substantial job-to-job transitions taking place in the aggregate labor market. Second, it contributes to explaining the volatility of vacancies due to the complementarity between vacancies and employed searchers that is present in the model.¹⁴

3.3. Sensitivity analysis. In this section, I discuss the sensitivity of the above results to my choice of model parameters. A key parameter of the model is the hiring cost. In Figure 5, I report the model-implied volatility of the job-finding and of the vacancy rate as a function of the hiring cost (expressed as a multiple of quarterly flow profits). To highlight the role of on-the-job search, I perform this exercise for two models: that with on-the-job search (corresponding to the first column of Table 2 when $H = 2.9\pi$) and that without on-the-job search (corresponding to the fourth column of Table 2 when $H = 2.9\pi$). Without a hiring cost, the introduction of on-the-job search reduces the model-implied volatility of the job-finding rate, though not that of the vacancy rate. In the presence of on-the-job search, an increase in the hiring cost has a significantly larger impact on

¹³For this exercise, I set μ equal to its mean value, recalibrate κ to maintain $s_u = 1$, which slightly increases the volatility of s_u and thereby of the job-finding rate, and set $\nu = 0.28$.

¹⁴This version of the model still explains more of the observed volatility than the model studied by Shimer. This is partly due to the higher estimate of b and to the wage-setting protocol assumed. When wages are set according to Equation (1), a drop in the job-finding rate in a recession does not have a strong negative feedback to the wage, eliminating a countervailing incentive to create relatively more vacancies during bad times.

the model-implied volatility of both the job-finding rate and the vacancy rate. For example, the introduction of a hiring cost of the calibrated magnitude increases the volatility of the job-finding and vacancy rates by 32% and 75%, respectively, without on-the-job search and by 89% and 174%, respectively, with on-the-job search.

Next, I study how my results vary with ρ , the curvature parameter of the search cost function, σ , the dispersion parameter of the idiosyncratic component distribution, β , the share of net match product captured by workers, and b , the flow payoff from unemployment.¹⁵

	$\rho = 1$	$\rho = 1.35$	$\rho = 2$
quit rate	0.087	0.101	0.118
prob. of contacting employed searcher	64.8%	70.8%	77.3%
fraction of new hires previously employed	46.6%	50.1%	54.0%
average idiosyncratic component	0.051	0.055	0.060
implied volatility and correlation			
contact rate, λ	0.0930	0.0991	0.1083
job-finding rate, f	0.1406	0.1376	0.1371
unemployment rate, u	0.1896	0.1870	0.1869
vacancy rate, v	0.1846	0.2028	0.2307
quit rate, q	0.0385	0.0359	0.0350
corr(u,v)	-0.959	-0.962	-0.966
corr(u,q)	-0.768	-0.686	-0.616

TABLE 3. Sensitivity analysis with respect to the curvature of the search cost function, ρ .

Table 3 reports the equilibrium value of the relevant variables for three different values of ρ , together with the volatilities implied by the model for these parameter values.¹⁶ Variation in the curvature of the search cost function has a large impact on the predicted magnitude of the quit rate. A larger value of ρ makes the search cost function more elastic and thereby implies higher search effort by employed workers relative to unemployed workers. The corresponding higher quit rate, in turn, increases the probability of contacting an employed searcher, the fraction of new hires who were previously employed, and the average idiosyncratic component among the employed. An increase in the extent of on-the-job search leads to increased incentives to create vacancies and thereby increases the volatility of the vacancy and contact rates. A larger value of ρ reduces the

¹⁵When changing these parameters, I keep $\lambda = 1.355$ and reset κ to maintain $s_u = 1$. Given that κ scales the contact rate and the search effort function, this is equivalent to keeping κ the same and changing λ to keep the job-finding rate at 1.355.

¹⁶Christensen, Lentz, Mortensen, Neumann, and Werwatz (2005) estimate a value of $\rho = 1$ using Danish data implying that the role of job-to-job transitions is somewhat smaller in the Danish labor market.

volatility of the search effort of unemployed workers. Thus the effect of a higher ρ on the volatility of the job-finding and unemployment rates is smaller than its effect on the contact rate.

	$\sigma = 0.05$	$\sigma = 0.10$	$\sigma = 0.15$
quit rate	0.086	0.101	0.108
prob. of contacting employed searcher	65.7%	70.8%	73.3%
fraction of new hires previously employed	46.3%	50.1%	52.0%
average idiosyncratic component	0.025	0.055	0.087
implied volatility and correlation			
contact rate, λ	0.0912	0.0991	0.1044
job-finding rate, f	0.1291	0.1376	0.1431
unemployment rate, u	0.1788	0.1870	0.1925
vacancy rate, v	0.1827	0.2028	0.2158
quit rate, q	0.0386	0.0359	0.0352
corr(u,v)	-0.957	-0.962	-0.965
corr(u,q)	-0.770	-0.686	-0.645

TABLE 4. Sensitivity analysis with respect to the dispersion in the match payoff, σ .

Table 4 reports the equilibrium value of the relevant variables for three different values of σ , together with the volatilities implied by the model for these parameter values. More dispersion in the idiosyncratic component implies higher search effort on the job and, correspondingly, a higher quit rate. Again, an increase in the extent of on-the-job search leads to increased labor-market volatility in the model through its effect on the incentives to create vacancies.

	$\beta = 0.80$	$\beta = 0.90$	$\beta = 0.95$
quit rate	0.103	0.101	0.099
prob. of contacting employed searcher	71.6%	70.8%	70.5%
fraction of new hires previously employed	50.7%	50.1%	49.9%
average idiosyncratic component	0.056	0.055	0.055
implied volatility and correlation			
contact rate, λ	0.1006	0.0991	0.0984
job-finding rate, f	0.1391	0.1376	0.1369
unemployment rate, u	0.1886	0.1870	0.1863
vacancy rate, v	0.2065	0.2028	0.2011
quit rate, q	0.0356	0.0359	0.0360
corr(u,v)	-0.963	-0.962	-0.962
corr(u,q)	-0.673	-0.686	-0.692

TABLE 5. Sensitivity analysis with respect to the worker's bargaining power, β .

Table 5 reports the equilibrium value of the relevant variables for three different values of β , together with the volatilities implied by the model for these parameter values. For these comparisons,

I always keep $H = 2.9\pi$; i.e., the hiring cost is always kept at its benchmark value as a fraction of flow profits. We have already seen that varying the ratio of the hiring cost to flow profits has a significant impact on the predictions of the model. The question that I address with this table is whether varying flow profits has any impact on my results when this ratio is kept constant. It is straightforward to see that what matters for firms' vacancy-creation decision is the ratio of the hiring cost to flow profits; given the calibration, this ratio does not change in response to changes in β . What does change is the wage paid to workers, which affects the search behavior of workers. A higher value of β implies a higher wage level and somewhat lower incentives to search for jobs with a high non-wage payoff, thereby decreasing the extent of on-the-job search in the model and thus reducing the model-implied volatilities. As can be seen in Table 5, the variation induced by this margin is quantitatively small, both in terms of levels (other than that of wages) and in terms of implied volatilities.

	$b = 0.71$	$b = 0.40$
calibrated value of ρ	1.35	1.81
implied volatility and correlation		
contact rate, λ	0.0991	0.0644
job-finding rate, f	0.1376	0.0812
unemployment rate, u	0.1870	0.1406
vacancy rate, v	0.2028	0.1252
quit rate, q	0.0359	0.0166
$\text{corr}(u,v)$	-0.962	-0.962
$\text{corr}(u,q)$	-0.686	-0.266

TABLE 6. Sensitivity analysis with respect to the flow payoff from unemployment.

Finally, Table 6 reports the equilibrium value of the relevant variables in my benchmark calibration and in a calibration that uses the more conservative $b = 0.4$ of Shimer. For this exercise, I vary the curvature parameter ρ to keep the quit rate at 0.101, since I already showed that varying the extent of on-the-job search affects the model-implied volatilities substantially. Just as in the simpler model reviewed in Mortensen and Nagypál (2007b), a lower value of b reduces the model-implied volatility. Even with Shimer's conservative estimate, however, the model succeeds in explaining 69%, 74%, and 62% of the observed volatility of the job-finding, unemployment, and vacancy rates, respectively.

4. INSPECTING THE MECHANISM

To get a better sense of the economic forces that give rise to the above results, in this section, I consider a simplified version of the above model. Instead of allowing for variable search effort, I normalize the search effort of unemployed workers to unity and assume that employed workers search with effort $s \leq 1$. I also assume that the model parameters are such that unemployed workers accept all matches, i.e., $\mu_r = \underline{\mu}$, just as in the calibrated equilibrium above.¹⁷ The advantage of this special case is that comparative statics around the steady state of the model can be fully characterized analytically. Moreover, this analysis can be done without any assumptions on the distribution function F . In this simplified model, what matters for worker transitions and, therefore, for job creation, is the rank of the idiosyncratic component μ in the distribution F , not its actual value. Since ranks are uniformly distributed, the independence from the functional form of F follows. The disadvantage of this special case is that it does not allow for a positive correlation between the search effort of workers and the probability that they accept a job offer. This aspect is important in the quantitative results reported above, since it alters the probability that a firm encounters workers at different points in the distribution of idiosyncratic values.

The positive selection of employed workers into idiosyncratic values toward the top of the distribution implies that, conditional on accepting a job, the expected turnover of previously unemployed workers is higher than that of previously employed workers.

Proposition 1. *The rate at which previously unemployed workers with tenure τ separate from their job is always higher than the same rate for previously employed workers with the same tenure.*

Proof. See Appendix. □

This result implies that the increase in the fraction of employed searchers $\gamma = \frac{s(1-u)}{u+s(1-u)}$ during times of low unemployment has two effects on the incentives to create vacancies. First, it decreases turnover conditional on match formation, thereby encouraging vacancy creation. Second, it decreases the probability that a match is formed, since employed searchers are less likely to agree to form a newly contacted employment match, thereby discouraging vacancy creation. A critical question is which of these two effects dominates. In other words, under what conditions does a

¹⁷Most empirical evidence (Devine and Kiefer (1991)) suggests that indeed unemployed searchers accept all matches.

firm have a higher expected payoff from contacting an employed searcher than from contacting an unemployed one?

Proposition 2. *In the steady state of the simplified economy, for given $r, \delta, s,$ and $\lambda,$ there exist*

$$0 < H_a < H_e < H_u$$

such that

- i) $J(\mu_r) \leq H$ if and only if $H \geq H_a,$*
- ii) $\Pi_e \geq \Pi_u$ if and only if $H \geq H_e,$ and*
- iii) $\Pi_u \geq 0$ if and only if $H \leq H_u.$*

Proof. See Appendix, which also specifies the expressions for $H_a, H_e,$ and $H_u.$ □

The relative expected payoff for a firm from contacting an employed versus an unemployed searcher depends crucially on the size of the hiring cost $H.$ When this cost is zero, the expected payoff from contacting unemployed searchers is larger than the payoff from contacting employed searchers. In this case, acceptance is always beneficial to the firm, since it has a positive payoff on *all* matches; thus the lower acceptance rate of employed searchers lowers the payoff from contacting them compared to the payoff from contacting unemployed searchers.

Once there is an initial cost of creating an employment relationship, however, it need not be true that firms have a positive expected payoff on *all* matches, even if their payoff is positive on average. A value of $H > H_a$ ensures that there is the possibility for a firm to have a negative payoff on *some* matches, since such a hiring cost implies that the expected payoff on a match with the worst accepted value, $\mu_r,$ is negative. Once $H > H_a,$ a firm has a negative expected payoff on all matches that have a high enough rate of job-to-job transitions, since the high turnover implies a low expected duration that does not allow the firm to recoup its initial investment. In particular, the expected payoff on a match is negative if μ is below a critical threshold, i.e., $\mu < \mu_c,$ where μ_c is increasing in $H.$ If a firm could distinguish such matches from those matches with a longer expected duration *before* paying the hiring cost, it would choose not to form them. Since the idiosyncratic

job amenity, μ , is not observable by the firm (at least not prior to paying the hiring cost), firms will create such matches as long as match creation has a positive payoff in expectation.¹⁸

The positive selection of workers into idiosyncratic values toward the top of the distribution that explains Claim 1 also implies that the high-turnover matches that a firm accrues a loss on when $H > H_a$ are exactly the matches with a low idiosyncratic component that unemployed searchers are more likely to accept. This reduces the payoff to a firm from contacting unemployed searchers. Proposition 2 states that, for a high enough hiring cost (i.e., for $H \geq H_e$), this effect is so large that firms have a lower expected payoff from contacting unemployed searchers.

To demonstrate further how a higher payoff from contacting employed searchers can arise for a large enough hiring cost, I plot in Figure 1 the payoff to a firm from creating matches of different idiosyncratic components and the probability that these different matches are accepted by unemployed and employed searchers. (For these calculations, I use the same parameter values that I used in the quantitative exercises in Section 3 and set $s = 0.4$.) The figure shows that it is precisely the matches that generate negative payoffs to the firm that employed searchers are likely to reject, while unemployed searchers accept jobs indiscriminately.

Proposition 2 defines a final threshold, H_u , above H_e . Once the hiring cost is above H_u , the expected payoff from contacting an unemployed worker becomes negative. At values of H above this threshold, an equilibrium exists only if it is assumed that firms cannot observe the current employment status of workers upon meeting them. Otherwise, they would reject hiring the unemployed workers, which could not be an equilibrium for a positive job-finding rate. Below this threshold, the assumption on the observability of the employment status of workers is irrelevant, since firms have a positive payoff from contacting both employed and unemployed workers. In all of my quantitative exercises, I set H below H_u .¹⁹

¹⁸The optimal allocation in this environment would demand that the hiring cost be paid by the worker, the informed party, and not the firm. Given that it is the firm that controls the hiring technology (provides job-specific training, for example), implementing the optimal allocation in a decentralized equilibrium would require the firm to commit to a contract in which the worker transfers to the firm upfront the hiring and recruiting costs and receives her marginal product thereafter. Such a contract is not viable if 1) workers do not have access to sufficient amount of borrowing, 2) the firm cannot commit to future wages so that competing firms could attract away applicants by offering contracts without an up-front payment, or 3) if there is an incentive for the firm to form potentially unproductive relationships simply to collect the transfer from the worker without actually expending the cost of appropriately training the worker.

¹⁹Propositions 1 and 2 hold in the general model studied in Section 2. It is only the algebra that becomes more tedious.

Finally, note that the results in Proposition 2 are for a fixed value of the job-finding rate λ . The results here should, therefore, be interpreted as a comparison of two economies with different hiring costs, but with the same extent of on-the-job search, discount rate, job-destruction rate, and job-finding rate.

Next, I study the response of the above economy to changes in aggregate driving forces. Throughout, I consider the steady state of the simplified model and rely on comparisons of steady states to assess the response of the model to changes in its parameters.

Proposition 3. *Across steady states in the simplified model, the elasticity of the job-finding rate with respect to labor productivity is*

$$\varepsilon_{\lambda p} = \frac{p}{p-b} g_{\lambda} (r, \delta, \lambda, s, \bar{H}, \nu),$$

where $\varepsilon_{xy} = \frac{d \ln x}{d \ln y}$ and $\bar{H} = \frac{H}{p-w}$, here and in the rest of the paper.

In addition, the elasticities of the unemployment rate, the vacancy rate, and the quit rate with respect to labor productivity are, respectively,

$$\begin{aligned} \varepsilon_{up} &= g_u(\delta, \lambda) \varepsilon_{\lambda p} \\ \varepsilon_{vp} &= \frac{\varepsilon_{\lambda p}}{\nu} + g_v(\delta, \lambda, s) \varepsilon_{\lambda p} \\ \varepsilon_{qp} &= g_q(\delta, \lambda, s) \varepsilon_{\lambda p}. \end{aligned}$$

Given the assumptions about the model parameters, $\varepsilon_{\lambda p} > 0$, $\varepsilon_{up} < 0$, $\varepsilon_{vp} > 0$, and $\varepsilon_{qp} > 0$.

Proof. See Appendix, which also specifies the functional form for g_{λ} , g_u , g_v , and g_q . □

The elasticity of the job-finding rate with respect to labor productivity, $\varepsilon_{\lambda p}$, can be expressed as the product of the elasticity of the firm's flow profit margin with respect to labor productivity, which, given the wage in Equation 1, is $\frac{p}{p-b}$, and a second term $g_{\lambda}(r, \delta, \lambda, s, \bar{H}, \nu)$, which captures the impact of on-the-job search and of the hiring cost. The same decomposition can be done for the elasticity of the unemployment rate, of the vacancy rate, and of the quit rate. This means that the impact of labor productivity shocks on the relevant labor-market variables can be decomposed into an effect coming through the wage-setting mechanism and an effect coming through turnover.

This decomposition is useful given the controversies in the literature about the appropriate way to model wage-setting in matching models. (For a discussion, see Mortensen and Nagypál (2007b).)

The relationship between the elasticity of the unemployment rate and that of the job-finding rate is the same in a model with on-the-job search as it is in one without it; i.e., g_u does not depend on s . This is not the case for vacancies, however. Once on-the-job search is introduced, market tightness is no longer equal to the ratio of vacancies to unemployment. In particular, the vacancy-unemployment ratio is the product of market tightness and the ratio of total searchers to unemployed searchers, which is a procyclical variable. This implies that by definition, in a model of on-the-job search, the vacancy-unemployment ratio is more procyclical than is market tightness. It also means that the ratio of the elasticity of the vacancy rate to the elasticity of the job-finding rate increases with the amount of on-the-job search; i.e., $\frac{\partial g_v}{\partial s} > 0$. This is an important result, since as we have seen in Section 3, the presence of on-the-job search makes the largest contribution toward explaining the volatility of the vacancy rate.

In determining the relationship between movements in the quit rate and the job-finding rate, there are two effects to consider. First, the quit rate for a worker with a given idiosyncratic component increases with the job-finding rate. Second, across steady states, a higher job-finding rate results in a shift of workers toward higher idiosyncratic components, which then results in a decrease in the quit rate. The proposition implies that the first effect always dominates, so that the quit rate unambiguously increases in response to an increase in productivity.

Proposition 4. *The effect of on-the-job search on the elasticity of the job-finding and vacancy rates with respect to labor productivity is more positive the larger is the hiring cost; i.e.,*

$$\frac{\partial^2 \varepsilon_{\lambda p}}{\partial s \partial \bar{H}} > 0 \quad \text{and} \quad \frac{\partial^2 \varepsilon_{vp}}{\partial s \partial \bar{H}} > 0,$$

if \bar{H} is sufficiently large.

Proof. See Appendix. □

This proposition implies that the complementarity between on-the-job search and the hiring cost in generating a large model response to changes in productivity is not only a characteristic of the benchmark model calibrated in Section 3, but is also a general property of this class of models. The intuition for the complementarity can be understood from the results of Proposition 2. Recall from

that proposition that, as H increases, the relative payoff from contacting an employed searcher rises. This result in turn means that the shift in the pool of searchers toward employed workers during an upturn raises the payoff from opening a vacancy, an effect that encourages vacancy creation. Since the extent of on-the-job search increases in s , this vacancy-creation effect is more pronounced for higher values of s . To put it differently, given the positive selection of workers into jobs with a high idiosyncratic component, employees hired from another job have a higher expected duration over which the cost of hiring can be recouped than do employees hired from unemployment. This finding means that the effective cost of hiring is declining in the fraction of new employees hired from another job, which becomes more procyclical the larger is s .

To demonstrate the extent of this complementarity, Figure 2 plots the elasticity of the job-finding rate and the vacancy rate with respect to labor productivity for different values of s and H , using the same values for the other parameters as above. The values of H that I consider are from zero to five times the quarterly flow profit of firms. In panel (a), the value of $\frac{p}{p-b} \frac{\nu}{1-\nu} = 2.30$, corresponding to $s = 0$ and $H = 0$, represents the model without any hiring cost or on-the-job search. When $H = 0$, the elasticity predicted by the model decreases in s . The decline, though, is relatively mild for the chosen parameter values, with the elasticity becoming 1.42 when $s = 1$. It can be shown that this decline in the elasticity of the job-finding rate when $H = 0$ is a general result that holds for any choice of parameters. The reason for this decline is apparent from Proposition 2: when there are no hiring costs, it is always more profitable to contact an unemployed worker so the presence of employed searchers dampens the variation in the incentives to create vacancies.²⁰ Notice from panel (b) that this decline is not present for the elasticity of the vacancy rate.

It is clear from Figure 2 that adding a hiring cost to the model without any on-the-job search increases the elasticity of the job-finding and vacancy rates with respect to labor productivity, as we already saw in Section 3. This increase is more pronounced when on-the-job search is present, especially for vacancies, because of the complementarity discussed in Proposition 4. Given the assumption that the previous employment status of searchers is observable to firms upon contact, a limit can be placed on the amount of amplification predicted by this simplified model. For a large amount of on-the-job search and a large hiring cost, firms have a negative expected payoff when contacting an unemployed searcher, since unemployed searchers have low expected duration. The

²⁰This also seems to be the case for the model studied by Pissarides (1994).

thick line labeled $H = H_u$ in Figure 2 plots the predicted elasticity as a function of s when the hiring cost is set so that a firm's expected payoff from contacting an unemployed searcher is exactly zero, i.e., $\Pi_u = 0$. Below and to the left of this curve lie points where the equilibrium features $\Pi_u > 0$. Notice that this limit is less binding in the more general model with variable search effort examined above. There, workers high in the idiosyncratic distribution that provide the highest payoff to firms choose a low search effort and thus have lower turnover than with a fixed search effort. In particular, $\Pi_u > 0$ in all the quantitative exercises of Section 3.

Next, I turn examine the response of this simplified economy to changes in the job-destruction rate.

Proposition 5. *Across steady states in the simplified model, the elasticity of the job-finding rate with respect to the job-destruction rate is*

$$\varepsilon_{\lambda\delta} = h_\lambda(r, \delta, \lambda, s, \bar{H}, \nu).$$

In addition, the elasticities of the unemployment rate, the vacancy rate, and the quit rate with respect to the job-destruction rate are, respectively,

$$\begin{aligned}\varepsilon_{u\delta} &= -g_u(\delta, \lambda)(1 - \varepsilon_{\lambda\delta}) \\ \varepsilon_{v\delta} &= \frac{\varepsilon_{\lambda\delta}}{\nu} - g_v(\delta, \lambda, s)(1 - \varepsilon_{\lambda\delta}) \\ \varepsilon_{q\delta} &= 1 - g_q(\delta, \lambda, s)(1 - \varepsilon_{\lambda\delta}).\end{aligned}$$

Given the assumptions on the model parameters, $\varepsilon_{\lambda\delta} < 1$, $\varepsilon_{u\delta} > 0$, $\varepsilon_{v\delta} < \frac{1}{\nu}$, and $\varepsilon_{q\delta} < 1$.

Proof. See Appendix, which also specifies the functional form for h_λ . □

In contrast with the model without on-the-job search, the elasticity of the job-finding rate with respect to the job-destruction rate is not necessarily negative. In the presence of on-the-job search, one effect of a higher job-destruction rate is to remove workers from jobs at the very top of the idiosyncratic component distribution and thereby increase the acceptance rate of employed searchers, encouraging vacancy creation. This effect has the potential to be strong enough to increase the ratio of vacancies to searchers, thereby increasing the job-finding rate. The proposition does show,

however, that this positive effect on the job-finding rate is never large enough to cause the unemployment rate to respond negatively to an increase in the job-destruction rate. Below I also show that for empirically relevant parameter values this positive effect on the job-finding rate is not strong enough to cause the elasticity of the job-finding rate to be positive.

The elasticity of vacancies is the sum of two terms. The first term is simply a multiple of the elasticity of the job-finding rate; thus, it has the same sign, which is generally negative. The second term is always positive and decreases in s for a given $\varepsilon_{\lambda\delta}$. When $s = 0$, the second term is equal to $(1 - u)(1 - \varepsilon_{\lambda\delta})$. For an $\varepsilon_{\lambda\delta}$ that is negative, but close to zero as in the standard model with empirically relevant parameter values, the second term is much larger than the first, so $\varepsilon_{v\delta} \approx 1 - u > 0$. In other words, the vacancy rate co-moves positively with the job-destruction rate and hence also with the unemployment rate, which is greatly at odds with the data, as pointed out by Shimer (2005). The reason for this result in the standard model is that an increase in the job-destruction rate increases the number of searching workers, thus encouraging vacancies to enter the market, even as the vacancy-unemployment ratio declines. Once on-the-job search is added to the model, however, the impact of an increase in the job-destruction rate on the number of searchers does decrease, and the weight on the second term declines. When $s = 1$, a change in the job-destruction rate has no impact on the number of searchers in the economy, and the second term in the expression for $\varepsilon_{v\delta}$ vanishes; this implies that the elasticity of the vacancy rate has the same sign as the elasticity of the job-finding rate, which is generally negative. Adding on-the-job search thus brings back changes in the job-destruction rate as a source of labor-market fluctuations in matching models.

The elasticity of the quit rate with respect to the job-destruction rate is not necessarily negative for the same reasons that the elasticity of the job-finding rate is not necessarily negative: a higher job-destruction rate moves workers toward lower idiosyncratic values in the distribution, which then increases the acceptance rate and, therefore, the quit rate of workers. Unlike in the case of the job-finding rate, this effect can be strong enough to cause the elasticity of the quit rate to be positive, even for empirically relevant parameter values. This explains the small and occasionally countercyclical response of the quit rate in Section 3 across steady states.

Proposition 6. *The effect of on-the-job search on the elasticity of the job-finding and vacancy rates with respect to the job-destruction rate is more negative the larger the hiring cost, i.e.,*

$$\frac{\partial^2 \varepsilon_{\lambda\delta}}{\partial s \partial \bar{H}} = \frac{\partial^2 h_{\lambda}(r, \delta, \lambda, s, \bar{H}, \nu)}{\partial s \partial \bar{H}} < 0 \quad \text{and} \quad \frac{\partial^2 \varepsilon_{v\delta}}{\partial s \partial \bar{H}} < 0,$$

if \bar{H} is sufficiently large and r is sufficiently small.

Proof. See Appendix. □

For a large enough hiring cost, there is again a complementarity between on-the-job search and the hiring cost, where the reason for this complementarity is the same as in the case of labor productivity. To demonstrate the extent of this complementarity, in Figure 3, I plot the elasticity with respect to the job-destruction rate of the job-finding rate and that of the vacancy rate for different values of s and H . I again use the same values for the other parameters as in Section 3. For these values, the model predicts an empirically correct sign for the elasticity of the job-finding rate. The model without on-the-job search or a hiring cost ($s = 0$ and $H = 0$) corresponds to a moderate elasticity of the job-finding rate and an elasticity of the vacancy rate that is close to 0. The elasticity of the job-finding rate declines in absolute value in s when $H = 0$, which again can be shown to be a general property of the simplified model. This decline is very significant, as the elasticity is -0.60 when $s = 0$ and is -0.01 when $s = 1$. Once again, it is clear from Figure 3 that adding a hiring cost to the model increases the magnitude of the elasticity of the job-finding rate with respect to the job-destruction rate, for the same reasons as in the case of labor productivity. The adding of a hiring cost also increases the absolute value of the elasticity of the vacancy rate somewhat. Further, Figure 3 clearly demonstrates that the complementarity discussed in Proposition 6 is again quantitatively relevant. For a hiring cost above two quarters' of profits, the elasticity of the job-finding rate increases with s over most of its range, although a hiring cost of three to four quarters' of profits is again required to generate a quantitatively large increase in the elasticity with s . Once more, there is a limit to how much amplification can be generated by the complementarity between the hiring cost and on-the-job search. The thick line labeled $H = H_u$ in Figure 3 plots the predicted elasticity as a function of s when the hiring cost is set to make a firm's expected payoff from contacting an unemployed worker exactly zero, i.e., $\Pi_u = 0$. Above and to the left of this curve lie points where the equilibrium features $\Pi_u > 0$.

5. EMPIRICAL EVIDENCE

5.1. Job switching and previous employment status. A key component of the proposed model is that newly employed workers who have been previously unemployed have a higher chance of switching jobs than newly employed workers who have been previously employed. To show that this is indeed the case, I consider a sample of individuals ages 25-60 from the Current Population Surveys from 1994 to 2004, who were observed for two consecutive months and who were hired into a new job in the first month and continue to be employed in the second month, though not necessarily by the same employer. (For a detailed description of the data construction, see Nagypál (2005b).) I estimate a logit specification for the probability of switching employers between the two months controlling for a worker's sex, race, age, education, and marital status, their job's industry and full-time status, and month and year effects. I find that previously unemployed workers have a 16.2% higher chance of switching employers than do previously employed workers (with a standard error of 3.2%). Once I restrict the sample to workers who self-report their own labor-market status, the effect becomes 19.5% (with a standard error of 5.1%).²¹ Due to the short panel component of the Current Population Survey, it is not possible to estimate job-switching probabilities at higher tenures, but these results certainly suggest that the expected time on a job before switching to another job is lower for previously unemployed workers.

5.2. Job-destruction shocks. In this paper, I argue that taking into account job-destruction shocks is important when accounting for the business-cycle volatility of key labor-market variables. Indeed, in the benchmark model, a significant fraction of the overall response is due to changes in the job-destruction rate: shutting down the volatility in the job-destruction rate would decrease the model-implied volatility of the job-finding rate by 24%, of the unemployment rate by 48%, and of the vacancy rate by 22%.

The theoretical possibility that, in the presence of on-the-job search, variation in the job-destruction rate need not induce a positive correlation between the unemployment and vacancy rates has been discussed above. This brings back job-destruction shocks as a *potential* driving force into matching models. To assess how relevant job-destruction shocks are as an *actual* driving force, notice that

²¹I have 104,689 observations over the ten years, with 41,592 self-reports.

these shocks impact the model-implied volatilities through two channels. First, volatility in the separation rate has a direct impact on the volatility of the unemployment rate. To the extent that a third of the variation in the unemployment rate is due to variation in the job-destruction/separation rate margin, taking this volatility into account is necessary to account fully for the variation in unemployment (and explains why the volatility of the unemployment rate declines the most when job-destruction shocks are shut down). Second, volatility in the separation rate affects the incentives to create vacancies in the model by changing the expected duration of *newly created* employment matches. Does a higher separation rate indeed signal a lower expected duration of new matches in the data? There is evidence that indeed it does: Bowlus (1995) and Davis, Haltiwanger, and Schuh (1996) both offer evidence that jobs created during recessions are more likely to be destroyed than jobs created during booms. Of course, to the extent that the volatility of the separation rate is partly explained by a burst of job destruction at the beginning of downturns (as argued by Davis, Haltiwanger, and Schuh (1996) and more recently by Fujita and Ramey (2007a)) which does not reflect changes in the expected duration of new matches, the calculations in Table 2 taking into account all the variation in the job-destruction rate could be somewhat overestimating the variation in the incentives to create vacancies and the resulting volatility of the job-finding and vacancy rates.²²

6. RELATIONSHIP TO EXISTING LITERATURE

The model I construct builds on the matching framework developed by Diamond (1982), Mortensen (1982), and Pissarides (1985) (see Pissarides (2000) for a review). In the last two decades, matching models have gained wide popularity in the analysis of aggregate labor markets due to their ability to explain several labor-market phenomena that the neoclassical growth model cannot tackle, such as the existence of equilibrium unemployment. Since the development of the model, several

²²Matching models with endogenous destruction are designed to explain the initial burst of job destruction that takes place in a recession as opposed to the drop in the expected duration of newly created matches. These models do not generate substantially more volatility than the textbook model with exogenous separations (see Mortensen and Nagypál (2007a)). Taking endogenous job-destruction into account in the presence of on-the-job search, a natural framework for which is the model of Nagypál (2005a), could nonetheless be important given the potential promise such an extension has for better explaining the cyclical behavior of the quit rate.

authors have asserted that it can quantitatively explain the variation in key labor-market variables over the business cycle.²³ Recently, however, this view has been challenged, and the standard matching model has been criticized for its lack of amplification when compared with data. Shimer (2005) shows that in response to shocks to the productivity of employment relationships and to the job-destruction rate, the textbook matching model predicts the volatility of the vacancy and unemployment rates to be an order of magnitude lower than the volatility observed in the data, assuming reasonable parameter values. The recent and active literature spurred by Shimer’s work, starting with Hall (2005), has mostly focused on the role of less responsive wages in resolving this “recruitment volatility puzzle”.²⁴ While there have been partial successes in bringing the model in line with the data (see Mortensen and Nagypál (2007b), in particular, who also give a detailed review of the literature), there has remained a substantial gap between what the model can explain and what is observed in the data. Recently, Haefke, Sonntag, and van Rens (2007) and Pissarides (2007) have questioned whether there is enough stickiness empirically in the relevant measure of wages to explain the observed volatility. All the recent papers addressing the “recruitment volatility puzzle” have abstracted from on-the-job search, with the exception of Krause and Lubik (2006) discussed below.

While the focus on the “recruitment volatility puzzle” and the interaction of on-the-job search and costly hiring is novel, my paper is not the first one to study the cyclical implications of matching models with on-the-job search. Pissarides (1994) and Mortensen (1994) both generalize a matching model by incorporating on-the-job search. Pissarides (1994) informally speculates that allowing for search by the employed reduces the responsiveness of unemployment to productivity shocks. Without a hiring cost, this is also true in my model. The reasons for such a result in his model are different than in mine, however. In particular, he develops a model with two type of jobs: bad jobs that are only taken by unemployed workers and good jobs that are taken both by unemployed and employed workers. The costs of creating these two types of jobs are set up so that the relative number of good jobs rises in response to an increase in aggregate labor productivity. With these good jobs being harder to get for the unemployed due to competition from employed searchers,

²³See, for example, Blanchard and Diamond (1989), Mortensen and Pissarides (1994), Andolfatto (1996), Merz (1995), Cole and Rogerson (1999), and Ramey and Watson (1997).

²⁴See Kennan (2007), Menzio (2005b), Hall and Milgrom (2007), Hagedorn and Manovskii (2007), and Gertler and Trigari (2006).

his conjecture follows. Given the structure of his model though, the acceptance rate of employed searchers is always unity, so the channels present in my framework with a disperse distribution of job values do not arise in his model. In the setup of Mortensen (1994), whose aim is to study the cyclical properties of job flows implied by his model, all newly created jobs have the highest payoff among all jobs, so again the issue of whether to accept a job or not does not arise.

More recently, Krause and Lubik (2006) study a model with on-the-job search and again two types of jobs: bad jobs that are easy to get and good jobs that are difficult to get in equilibrium. They assume that search by workers can be directed toward the two types of jobs, which thus creates two separate markets. While the job-finding rate in each market varies over the business cycle no more than in the textbook matching model, a feature of their model is that unemployed workers apply for more bad jobs in a boom while they wait to get good jobs in a recession. This feature is present because the authors make the critical, but questionable, assumption that the search technology available to employed workers is elastic while that available to unemployed workers is inelastic. This makes unemployed workers want to take bad jobs more in a boom because the on-the-job search technology becomes more valuable compared to the value of unemployed search. Given that the job-finding rate for bad jobs is higher, this composition effect leads to an increased volatility of the aggregate job-finding rate.

The setup of Barlevy (2002) is the most similar to the model studied in this paper, although in his model it is heterogeneity in productivity that motivates on-the-job search while in my model, it is heterogeneity in job amenity. He argues that recessions are times when the reallocation of employed workers toward their most productive use is hindered by what he calls the “sullyng effect”. To the extent that the distribution of job amenities shifts downwards in a recession, such a sullyng effect is present in my model as well. The results of this paper are novel, since unlike Barlevy (2002), I focus on the amount of amplification generated by the model, derive analytical comparative static results, and discuss the role of the complementarity between hiring costs and on-the-job search in generating fluctuations.

7. CONCLUSION

This paper has shown that allowing for on-the-job search and a positive reinforcement between recruiting activity and search by employed workers is critical to explain the observed large cyclical

volatility of labor-market aggregates. When workers are searching on the job, firms that contemplate advertising a vacant position need to consider both the probability that a contact with a searching worker results in an employment relationship and the likelihood that a recruited worker will quit in the future. In the presence of a moderate hiring cost, firms expect higher profits from contacting workers who are less likely to quit, even if these workers have a lower probability of accepting a job. Employed searchers are exactly such workers: they are less likely to accept a job offered to them, but if they do accept, they are less likely to quit due to the positive selection of employed workers into better and better employment matches.

When the payoff from contacting employed searchers is higher, the presence of more vacancies and more employed searchers enhance each other in a boom. This creates more amplification in a model that takes into account on-the-job search compared to one that abstracts from it. In addition, in the presence of on-the-job search, changes in the job-destruction rate do not induce a counterfactual positive correlation between the unemployment and vacancy rates, thereby bringing back changes in the job-destruction rate as a driving force behind labor-market fluctuations.

The mechanism of the model studied here is not the only mechanism capable of generating positive reinforcement between recruiting activity and on-the-job search in the context of a random matching model. Another class of models can give rise to such reinforcement based on the simple idea that workers are heterogeneous in the payoff they generate to firms (due to differences in productivity or differences in turnover rates). If there is a positive correlation between the probability of employment and the payoff a worker generates to an employer, this could also give rise to a higher payoff from contacting employed searchers. While intuition would suggest that such a correlation is certainly present for older and highly educated workers, it is difficult to argue that such a correlation could explain all the labor-market volatility of less educated and younger workers.

Incorporating on-the-job search and its interaction with recruiting is important not only in the study of cyclical volatility. The results of this paper suggest that this interaction is also important in understanding the response of the aggregate labor market to policy interventions, such as the provision of unemployment insurance. Moreover, the model of this paper offers a novel framework to use to study the cost of business cycles in the presence of search externalities.

8. APPENDIX

8.1. The wage bargain. I derive the wage outcome by taking the limit of the equivalent discrete-time bargaining problem. Assume that time is discrete in increments of Δ . Timing in the period $[t, t + \Delta)$ is as follows:

- (1) workers and firms arrive to a period either as matched (with a single partner) or unmatched
- (2) matched worker-firm pairs bargain over the wage for the period
- (3) workers decide how much to search during the period
- (4) production (in relationships that reached an agreement over the wage) and search takes place during the period
- (5) at the end of the period, existing relationship break down with probability $\delta\Delta e^{-\delta\Delta}$
- (6) workers who searched during the period and firms participate in the matching process
- (7) if a worker or a firm matches with a new partner, they have to decide whether to continue their existing relationship (or unemployment or vacancy) or end that relationship and form the new one

The bargaining between the worker and the firm over the flow wage prevailing during the period $[t, t + \Delta)$ takes place as follows. (The wage is constant during the period. As the length of the period goes to 0, this assumption becomes unrestrictive.) If the worker and the firm agree on a wage, production takes place, and the worker enjoys utility $w + \mu$ and devotes s_w effort to searching for a better job during the period. If the worker and the firm fail to agree on a wage today, the worker enjoys utility $b + \mu$ and devotes s_d effort to searching for a better job during the period, while the firm has a payoff of 0 during the period. In case of disagreement, the worker and firm can resume negotiations at the beginning of the next period. If a worker searches during period $[t, t + \Delta)$ with effort s , it contacts a new job opportunity during the matching phase with probability $s\lambda_{t+\Delta}\Delta e^{-s\lambda_{t+\Delta}\Delta}$ from the distribution $F_{t+\Delta}(W')$.

The payoff of the worker in case of agreement on flow wage w_t is

$$W_t = \max_{s_w} \left\{ (w_t + \mu - k(s_w)) \Delta + e^{-r\Delta} (1 - \delta\Delta e^{-\delta\Delta}) \left(s_w \lambda_{t+\Delta} \Delta e^{-s_w \lambda_{t+\Delta} \Delta} \int \max(0, W' - N_{t+\Delta}) dF_{t+\Delta}(W') + N_{t+\Delta} \right) \right. \\ \left. + e^{-r\Delta} \delta\Delta e^{-\delta\Delta} \left(s_w \lambda_{t+\Delta} \Delta e^{-s_w \lambda_{t+\Delta} \Delta} \int \max(0, W' - U_{t+\Delta}) dF_{t+\Delta}(W') + U_{t+\Delta} \right) \right\},$$

where $N_{t+\Delta}$ is the expected payoff from the relationship from time $t + \Delta$ onwards. The choice of s_w is independent of w_t , hence the worker's search decision is not influenced by w_t . The payoff of the worker in case of disagreement is

$$D_t^w = \max_{s_d} \left\{ (b + \mu - k(s_d)) \Delta + e^{-r\Delta} (1 - \delta\Delta e^{-\delta\Delta}) \left(s_d \lambda_{t+\Delta} \Delta e^{-s_d \lambda_{t+\Delta} \Delta} \int \max(0, W' - N_{t+\Delta}) dF_{t+\Delta}(W') + N_{t+\Delta} \right) \right. \\ \left. + e^{-r\Delta} \delta\Delta e^{-\delta\Delta} \left(s_d \lambda_{t+\Delta} \Delta e^{-s_d \lambda_{t+\Delta} \Delta} \int \max(0, W' - U_{t+\Delta}) dF_{t+\Delta}(W') + U_{t+\Delta} \right) \right\}.$$

The payoff of the firm in case of agreement on flow wage w_t is

$$J_t = (p - w_t) \Delta + e^{-r\Delta} (1 - \delta\Delta e^{-\delta\Delta}) \left(s_w \lambda_{t+\Delta} \Delta e^{-s_w \lambda_{t+\Delta} \Delta} (1 - F_{t+\Delta}(N_{t+\Delta})) (V_{t+\Delta} - J_{t+\Delta}) + J_{t+\Delta} \right) \\ + e^{-r\Delta} \delta\Delta e^{-\delta\Delta} V_{t+\Delta}.$$

The payoff of the firm in case of disagreement is

$$D_t^f = e^{-r\Delta} (1 - \delta\Delta e^{-\delta\Delta}) \left(s_d \lambda_{t+\Delta} \Delta e^{-s_d \lambda_{t+\Delta} \Delta} (1 - F_{t+\Delta}(N_{t+\Delta})) (V_{t+\Delta} - J_{t+\Delta}) + J_{t+\Delta} \right) + \\ + e^{-r\Delta} \delta\Delta e^{-\delta\Delta} V_{t+\Delta}.$$

Notice that the worker makes the same search decision whether or not production takes place, so that $s_w = s_d$. Thus the difference in the worker's payoff per unit of time is $w_t - b$, while the difference in the firm's payoff per unit of time is $p - w_t$. The Nash outcome of the bargain in period t is then

$$w_t = b + \beta(p - b).$$

8.2. How good are steady-state approximations? As is well established, in the standard matching model without on-the-job search, comparative static exercises or log-linearization around the non-stochastic steady state invariably give results that are very close to the dynamic response of the full stochastic model, as long as the driving process is persistent enough (see Shimer (2005) and the proof in Mortensen and Nagypál (2007b)). In the standard model, transition dynamics are very fast, due to the forward-looking and instantaneous adjustment of the number of vacancies and thus of the job-finding rate and the calibrated high job-finding rate. Due to the presence of on-the-job search, the full stochastic version of my model is significantly more complex than that of the standard model: the state space includes not only the unemployment rate, but also the complete distribution of idiosyncratic values across employed workers. It is still true though

that there is no delay in the adjustment of vacancies due to the assumption of free entry. Hence, the dynamics of the dynamic stochastic model along the transition path are determined by the dynamics of the incentives to create vacancies. These, in turn, are determined by the dynamics of the two state variables. Since the unemployment rate adjusts just as quickly as in the standard model due to the high calibrated job-finding rate, along most of the transition path, the dynamics of vacancies is determined by the evolution of the distribution of employed workers. For this reason, given a persistent driving process, the calculated responses of Table 2 in the short run — when the distribution of employed workers is left unchanged — and in the long run — when the distribution of employed workers fully adjusts to its new steady-state value — give two useful bounds on the response of the full dynamic model.

A further important determinant of the similarity of the steady-state approximations to the full dynamic responses is the persistence of the driving process. Shimer (2005) reports the labor productivity process to be highly persistent, with a quarterly auto-correlation of 0.878. This means that using steady-state approximations gives almost identical results to his simulation of the full stochastic model (an implied volatility of market tightness of 0.034 and 0.035, respectively). Since the quarterly autocorrelation of the separation rate is somewhat smaller, 0.733, in this case using steady-state approximations gives slightly larger responses than his simulation of the full stochastic model (an implied volatility of market tightness of 0.007 and 0.006, respectively). This implies that my calculations, using steady-state responses, are likely to only slightly overstate the response of the full stochastic model.

8.3. Useful mathematical facts. Throughout the proofs, I use the following facts.

Fact 1. $\frac{\ln(1+y)}{y} < 1$ for $y > 0$ and is decreasing in y .

Fact 2. Let $n(y) = \frac{y}{(1+y)\ln(1+y)}$. $n'(y) < 0$ when $y > 0$ and $\lim_{y \rightarrow 0} n(y) = 1$.

Fact 3. Let $\hat{n}(a, y) = an(ay)$. $\frac{\partial \hat{n}(a, y)}{\partial a} > 0$ for $y > 0$.

Proof. By simple algebra. □

8.4. Proof of Proposition 1. The rate at which previously unemployed workers employed at tenure $\tau \geq 0$ separate from their job is

$$S_\tau^u = \delta + \lambda s \int_{\mu_r}^{\bar{\mu}} \bar{F}(\mu) dQ_\tau^u(\mu),$$

where Q_τ^u is the distribution of idiosyncratic values among previously unemployed workers at tenure τ . S_τ^e and Q_τ^e can be similarly defined. Then, in order to show that $S_\tau^e < S_\tau^u$, it is sufficient to show that Q_τ^e strictly first-order stochastically dominates Q_τ^u for all $\tau \geq 0$.

Given the acceptance decisions of workers, the above distributions at $\tau = 0$ can be expressed as

$$\begin{aligned} Q_0^u(\mu) &= \frac{F(\mu) - F(\mu_r)}{\bar{F}(\mu_r)} \\ Q_0^e(\mu) &= \frac{\int_{\mu_r}^{\mu} (F(\mu) - F(\mu')) dG(\mu')}{\int_{\mu_r}^{\bar{\mu}} \bar{F}(\mu') dG(\mu')}. \end{aligned}$$

For $\mu, \mu' \in [\mu_r, \bar{\mu})$, define $d(\mu, \mu') = \frac{F(\mu) - F(\mu')}{\bar{F}(\mu')}$. Given the assumptions on $F(\cdot)$, $d(\mu, \mu')$ is strictly decreasing in μ' since

$$\frac{\partial d(\mu, \mu')}{\partial \mu'} = \frac{-F'(\mu')\bar{F}(\mu') + F'(\mu')(F(\mu) - F(\mu'))}{\bar{F}(\mu')^2} = \frac{-F'(\mu')\bar{F}(\mu)}{\bar{F}(\mu')^2} < 0.$$

Then, for $\mu > \mu_r$,

$$Q_0^e(\mu) = \frac{\int_{\mu_r}^{\mu} d(\mu, \mu')\bar{F}(\mu')dG(\mu')}{\int_{\mu_r}^{\bar{\mu}} \bar{F}(\mu')dG(\mu')} < \frac{\int_{\mu_r}^{\mu} d(\mu, \mu_r)\bar{F}(\mu')dG(\mu')}{\int_{\mu_r}^{\bar{\mu}} \bar{F}(\mu')dG(\mu')} \leq d(\mu, \mu_r) = Q_0^u(\mu),$$

hence the initial distribution of the employed strictly first-order stochastically dominates the initial distribution of the unemployed.

As for the distributions Q_τ^u and Q_τ^e for $\tau > 0$, these can be derived from Q_0^u and Q_0^e by applying the same out-flow rates to them as a function of μ , which is $\lambda s \bar{F}(\mu)$. Since these rates do not depend on whether the worker was previously employed or unemployed, the resulting Q_τ^e strictly first-order stochastically dominates Q_τ^u .

8.5. Proof of Proposition 2. In the simplified model, equating the flows into and out of the relevant pools gives

$$(11) \quad u = \frac{\delta}{\delta + \lambda}$$

and

$$(12) \quad G(\mu) = \frac{u\lambda F(\mu)}{\delta + \lambda s \bar{F}(\mu)}.$$

The probability that a match of value μ is accepted by an employed searcher can be then expressed as

$$A_e(\mu) = \frac{G(\mu)}{G(\bar{\mu})} = \frac{\delta F(\mu)}{\delta + \lambda s \bar{F}(\mu)},$$

so that the payoff from contacting an employed worker can be written as

$$\begin{aligned} \Pi_e &= \int_{\underline{\mu}}^{\bar{\mu}} \left(\frac{p-w}{r+\delta+\lambda s \bar{F}(\mu)} - H \right) \frac{\delta F(\mu)}{\delta + \lambda s \bar{F}(\mu)} dF(\mu) = \\ &= \int_0^1 \left(\frac{p-w}{r+\delta+\lambda s(1-x)} - H \right) \frac{\delta x}{\delta + \lambda s(1-x)} dx = \\ &= (p-w)\delta \int_0^1 \frac{1}{r+\delta+\lambda s(1-x)} \frac{x}{\delta + \lambda s(1-x)} dx - H\delta \int_0^1 \frac{x-x_r}{\delta + \lambda s(1-x)} dx, \end{aligned}$$

where I used a change of variables $x = F(\mu)$. Then, given the definite integrals

$$\int_0^1 \frac{x}{\delta + \lambda s(1-x)} dx = \frac{1}{\lambda s} \left[\frac{\delta + \lambda s}{\lambda s} m_1(\delta, \lambda, s) - 1 \right]$$

and

$$\begin{aligned} \int_0^1 \frac{1}{r+\delta+\lambda s(1-x)} \frac{x}{\delta + \lambda s(1-x)} dx &= \\ &= \frac{1}{\lambda^2 s^2 r} [(\delta + \lambda s) m_1(\delta, \lambda, s) - (r + \delta + \lambda s) m_2(r, \delta, \lambda, s)], \end{aligned}$$

where

$$(13) \quad m_1(\delta, \lambda, s) = \ln(\delta + \lambda s) - \ln \delta$$

$$(14) \quad m_2(r, \delta, \lambda, s) = \ln(r + \delta + \lambda s) - \ln(r + \delta),$$

substituting in and simplifying gives

$$\Pi_e = \frac{p-w}{\lambda} \frac{\delta}{\lambda s r} \left(\frac{\delta + \lambda s}{s} m_1(\delta, \lambda, s) (1 - r\bar{H}) - \frac{r + \delta + \lambda s}{s} m_2(r, \delta, \lambda, s) + r\lambda\bar{H} \right).$$

Similarly, notice that the payoff from contacting an unemployed worker can be written as

$$\begin{aligned} \Pi_u &= \int_{\underline{\mu}}^{\bar{\mu}} \left(\frac{p-w}{r+\delta+\lambda s \bar{F}(\mu)} - H \right) dF(\mu) = \\ &= \int_0^1 \left(\frac{p-w}{r+\delta+\lambda s(1-x)} - H \right) dx = \frac{p-w}{\lambda} \left(\frac{m_2(r, \delta, \lambda, s)}{s} - \lambda\bar{H} \right). \end{aligned}$$

Statement i) of the proposition follows from simple substitution where

$$\bar{H}_a = \frac{1}{r + \delta + \hat{\lambda}s}.$$

Statement ii) follows from the fact that, for all \bar{H} ,

$$(15) \quad \frac{\partial \Pi_e}{\partial \bar{H}} - \frac{\partial \Pi_u}{\partial \bar{H}} = \frac{p-w}{\lambda} \frac{\delta + \lambda s}{s} \left(1 - \frac{\delta}{\lambda s} m_1 \right) > 0,$$

given Fact 1 of Section 8.3 applied to $y = \frac{\lambda s}{\delta}$. Similarly, statement iii) of the proposition follows from $\frac{\partial \Pi_u}{\partial \bar{H}} = -(p-w) < 0$ for all \bar{H} . The derivation of the expressions

$$\bar{H}_e(r, \delta, s, \lambda) = \frac{(r + \delta) m_2(r, \delta, \lambda, s) - \delta m_1(\delta, \lambda, s)}{r \lambda s - r \delta m_1(\delta, \lambda, s)}$$

and

$$\bar{H}_u(r, \delta, s, \lambda) = \frac{m_2(r, \delta, \lambda, s)}{\lambda s}$$

is straightforward, given that Π_e and Π_u are linear in \bar{H} . Finally, $\bar{H}_u > \bar{H}_e > 0$ follows from the expressions for Π_e and Π_u .

8.6. Proof of Proposition 3. Using the expressions for Π_e and Π_u derived above, we can rewrite the job-creation condition in the simplified model as

$$(16) \quad \frac{c}{\eta} = \frac{\delta}{\lambda r s} ((p - w - rH) m_1 - (p - w) m_2),$$

where again m_1 and m_2 are defined in Equations 13 and 14.

Given the assumptions on the matching function, notice that $-\frac{d \ln \eta}{d \ln p} = \frac{1-\nu}{\nu} \frac{d \ln \lambda}{d \ln p}$. Taking logs of Equation 16 and differentiating with respect to $\ln p$ gives

$$\begin{aligned} \frac{1-\nu}{\nu} \frac{d \ln \lambda}{d \ln p} &= -\frac{d \ln \lambda}{d \ln p} + \omega \left(\frac{\pi}{\pi - rH} \frac{d \ln \pi}{d \ln p} + \frac{\partial \ln m_1}{\partial \ln \lambda} \frac{d \ln \lambda}{d \ln p} \right) + \\ &+ (1-\omega) \left(\frac{d \ln \pi}{d \ln p} + \frac{\partial \ln m_2}{\partial \ln \lambda} \frac{d \ln \lambda}{d \ln p} \right), \end{aligned}$$

where

$$(17) \quad \omega = \frac{(p - w - rH) m_1}{(p - w - rH) m_1 - m_2}.$$

Substituting for the derivatives of m_1 and m_2 and collecting terms we get

$$\left(\frac{1}{\nu} - \frac{\omega}{m_1} \frac{\lambda s}{\delta + \lambda s} - \frac{1 - \omega}{m_2} \frac{\lambda s}{r + \delta + \lambda s} \right) \frac{d \ln \lambda}{d \ln p} = \left(\omega \frac{r \bar{H}}{1 - r \bar{H}} + 1 \right) \frac{d \ln \pi}{d \ln p}$$

so that

$$g_\lambda(r, \delta, \lambda, s, \bar{H}, \nu) = \frac{\omega(r, \delta, \lambda, s, \bar{H}) \frac{r \bar{H}}{1 - r \bar{H}} + 1}{l(r, \delta, \lambda, s, \bar{H}, \nu)},$$

where

$$(18) \quad l(r, \delta, \lambda, s, \bar{H}, \nu) = \frac{1}{\nu} - \frac{\omega(r, \delta, \lambda, s, \bar{H})}{m_1(\delta, \lambda, s)} \frac{\lambda s}{\delta + \lambda s} - \frac{1 - \omega(r, \delta, \lambda, s, \bar{H})}{m_2(r, \delta, \lambda, s)} \frac{\lambda s}{r + \delta + \lambda s}.$$

$g_\lambda > 0$ follows from the fact that l is strictly positive. To see this, note that we can write $l = l_1 + l_2 \omega$, where

$$\begin{aligned} l_1 &= \frac{1}{\nu} - n\left(\frac{\lambda s}{r + \delta}\right) > \frac{1 - \nu}{\nu} > 0 \\ l_2 &= n\left(\frac{\lambda s}{r + \delta}\right) - n\left(\frac{\lambda s}{\delta}\right) > 0, \end{aligned}$$

where the function $n(\cdot)$ is defined in Fact 2 of Section 8.3, and the inequalities follow from the properties of the function $n(\cdot)$ established there.

To derive ε_{up} , simply notice that

$$g_u(\delta, \lambda) = \frac{\partial \ln u}{\partial \ln \lambda} = -\frac{\lambda}{\delta + \lambda} < 0.$$

To derive ε_{vp} , notice that $v = \theta(u + s(1 - u))$, so that

$$\begin{aligned} \frac{\partial \ln v}{\partial \ln \lambda} &= \frac{\partial \ln \theta}{\partial \ln \lambda} + \frac{\partial \ln(u + s(1 - u))}{\partial \ln \lambda} = \frac{1}{\nu} + \frac{\partial \ln u}{\partial \ln \lambda} \frac{(1 - s)u}{u + s(1 - u)} = \\ &= \frac{1}{\nu} - \frac{\lambda}{\delta + \lambda} \frac{(1 - s)\delta}{\delta + s\lambda} \geq \frac{1}{\nu} - 1 > 0, \end{aligned}$$

thus $g_v(\delta, \lambda, s) = -\frac{\lambda}{\delta + \lambda} \frac{(1 - s)\delta}{\delta + s\lambda}$.

To derive ε_{qp} , first derive the quit rate for a general λ as

$$(19) \quad \begin{aligned} q &= \frac{\lambda s \int_{\underline{\mu}}^{\bar{\mu}} \bar{F}(\mu) dG(\mu)}{1 - u} = \lambda s \int_{\underline{\mu}}^{\bar{\mu}} \frac{G(\mu)}{1 - u} dF(\mu) = \\ &= \delta \lambda s \int_0^1 \frac{x}{\delta + \lambda s(1 - x)} dx = \delta \frac{\delta m_1 + \lambda s m_1 - \lambda s}{\lambda s}, \end{aligned}$$

where the first equality follows from integration by parts and the second equality uses the expressions for u and $G(\mu)$ in Equations(11) and (12) and a change of variables with $x = F(\mu)$. Then

$$(20) \quad \begin{aligned} g_q(\delta, \lambda, s) &= \frac{\partial \ln q}{\partial \ln \lambda} = \frac{\lambda}{\delta m_1 + \lambda s m_1 - \lambda s} \left((\delta + \lambda s) \frac{\partial m_1}{\partial \lambda} + s m_1 - s \right) - 1 = \\ &= \frac{\lambda s m_1}{\delta m_1 + \lambda s m_1 - \lambda s} - 1 = \frac{\lambda s - \delta m_1}{(\lambda s + \delta) m_1 - \lambda s} > 0, \end{aligned}$$

where the last inequality follows from the fact that g_q is decreasing in m_1 and $m_1 < \frac{\lambda s}{\delta}$.

8.7. Proof of Theorem 4. The derivation of the result regarding the cross-partial is algebraically tedious, so here I only sketch the steps of the proof. The first step is to notice that, as a function of s and \bar{H} , g_λ can be written as

$$g_\lambda = \frac{g_\lambda^1(s)}{g_\lambda^2(s) - g_\lambda^3(s)r\bar{H}},$$

where both the numerator and the denominator are positive and $g_\lambda^3(s) > 0$. Then, suppressing the dependence on s ,

$$\frac{\partial^2 g_\lambda}{\partial \bar{H} \partial s} = r \frac{(g_\lambda^{3'} g_\lambda^1 + g_\lambda^3 g_\lambda^{1'}) g_\lambda^2 - 2g_\lambda^{2'} g_\lambda^3 g_\lambda^1 + (g_\lambda^{3'} g_\lambda^1 - g_\lambda^3 g_\lambda^{1'}) g_\lambda^3 r \bar{H}}{(g_\lambda^2 - g_\lambda^3 r \bar{H})^3}$$

To show that $\frac{\partial^2 g_\lambda}{\partial \bar{H} \partial s} > 0$ for a sufficiently large \bar{H} , it is enough to show that $g_\lambda^{3'} g_\lambda^1 - g_\lambda^3 g_\lambda^{1'} > 0$. The next step is to note that

$$\begin{aligned} g_\lambda^{3'} g_\lambda^1 - g_\lambda^3 g_\lambda^{1'} &= \frac{1}{s\nu} \left(\ln(1+y_1) \frac{y_2}{1+y_2} - \ln(1+y_2) \frac{y_1}{1+y_1} \right) + \\ &+ \frac{y_1}{s(1+y_1)} \left[\frac{y_1}{1+y_1} - \frac{y_2}{1+y_2} - \frac{1}{1+y_1} (\ln(1+y_1) - \ln(1+y_2)) \right], \end{aligned}$$

where $y_1 = \frac{\lambda s}{\delta}$ and $y_2 = \frac{\lambda s}{r+\delta}$. Given Fact 2 of Section 8.3, this is decreasing in ν , so it is larger than the same expression evaluated at $\nu = 1$, thus

$$g_\lambda^{3'} g_\lambda^1 - g_\lambda^3 g_\lambda^{1'} \geq \frac{y_1}{s(1+y_1)} \left[\left(\frac{y_1}{1+y_1} - \frac{y_2}{1+y_2} \right) \left(1 - \frac{\ln(1+y_1)}{y_1} \right) + \frac{\ln(1+y_1) y_2}{1+y_2} - \frac{\ln(1+y_2) y_1}{1+y_1} \right].$$

This lower bound is positive, in turn, which follows again from Facts 1 and 2 of Section 8.3.

The statement regarding the cross-partial of ε_{vp} follows from the fact that

$$\frac{\partial^2 \varepsilon_{vp}}{\partial s \partial \bar{H}} = \frac{\partial^2 \varepsilon_{\lambda p}}{\partial s \partial \bar{H}} \left(\frac{1}{\nu} - \frac{\lambda}{\delta + \lambda} \frac{(1-s)\delta}{\delta + s\lambda} \right) + \frac{\partial \varepsilon_{\lambda p}}{\partial \bar{H}} \frac{\lambda \delta}{(\delta + s\lambda)^2}.$$

8.8. **Proof of Proposition 5.** To derive the expression for $\varepsilon_{\lambda\delta}$, notice that

$$\begin{aligned}\frac{d \ln m_1}{d \ln \delta} &= \frac{\partial \ln m_1}{\partial \ln \delta} + \frac{\partial \ln m_1}{\partial \ln \lambda} \frac{d \ln \lambda}{d \ln \delta} = \frac{\lambda s}{m_1 (\delta + \lambda s)} \left(\frac{d \ln \lambda}{d \ln \delta} - 1 \right) \\ \frac{d \ln m_2}{d \ln \delta} &= \frac{\partial \ln m_2}{\partial \ln \delta} + \frac{\partial \ln m_2}{\partial \ln \lambda} \frac{d \ln \lambda}{d \ln \delta} = \frac{\lambda s}{m_2 (r + \delta + \lambda s)} \left(\frac{d \ln \lambda}{d \ln \delta} - \frac{\delta}{r + \delta} \right),\end{aligned}$$

where m_1 and m_2 are defined in Equations 13 and 14. Taking logs of the job-creation condition in Equation (16) and differentiating the result with respect to $\ln \delta$ gives

$$\frac{1 - \nu}{\nu} \frac{d \ln \lambda}{d \ln \delta} = 1 - \frac{d \ln \lambda}{d \ln \delta} + \omega \frac{d \ln m_1}{d \ln \delta} + (1 - \omega) \frac{d \ln m_2}{d \ln \delta}.$$

Substituting in the values of $\frac{d \ln m_1}{d \ln \delta}$ and $\frac{d \ln m_2}{d \ln \delta}$ and collecting terms results in

$$h_\lambda(r, \delta, \lambda, s, \bar{H}, \nu) = \frac{l(r, \delta, \lambda, s, \bar{H}, \nu) - \frac{1-\nu}{\nu} - \frac{\omega(r, \delta, \lambda, s, \bar{H}) - 1}{m_2(r, \delta, \lambda, s)} \frac{r}{r+\delta} \frac{\lambda s}{r+\delta+\lambda s}}{l(r, \delta, \lambda, s, \bar{H}, \nu)},$$

where ω and l are defined in Equations (17) and (18).

While the denominator of h_λ is always positive as shown in the proof of Proposition 3, the sign of the numerator cannot be generally determined. In fact, numerical calculations show that the elasticity is positive for some admissible parameter values. Clearly, however, given the fact that $\nu \in (0, 1)$ and $\omega > 1$, the numerator of h_λ is smaller than its denominator, so $\varepsilon_{\lambda\delta} < 1$.

To derive ε_{up} , recall that, when $\hat{\lambda} = \lambda$, the unemployment rate is $u = \frac{\delta}{\delta + \lambda}$. Then

$$\frac{d \ln u}{d \ln \delta} = \frac{\partial \ln u}{\partial \ln \lambda} \frac{d \ln \lambda}{d \ln \delta} + \frac{\partial \ln u}{\partial \ln \delta} = \frac{\lambda(1 - \varepsilon_{\lambda\delta})}{\delta + \lambda}.$$

This is positive since $\varepsilon_{\lambda\delta} < 1$.

To derive ε_{vp} , notice that $v = \theta(u + s(1 - u))$ implies that

$$\begin{aligned}\frac{d \ln v}{d \ln \delta} &= \frac{d \ln \theta}{d \ln \delta} + \frac{d \ln(u + s(1 - u))}{d \ln \delta} = \frac{1}{\nu} \frac{d \ln \lambda}{d \ln \delta} + \frac{d \ln u}{d \ln \delta} \frac{(1 - s)u}{u + s(1 - u)} = \\ &= \frac{\varepsilon_{\lambda\delta}}{\nu} + \frac{\lambda}{\delta + \lambda} \frac{(1 - s)\delta}{\delta + s\lambda} (1 - \varepsilon_{\lambda\delta}).\end{aligned}$$

This expression is increasing in $\varepsilon_{\lambda\delta}$ since $\frac{1}{\nu} > 1 > \frac{\lambda}{\delta + \lambda} \frac{(1 - s)\delta}{\delta + s\lambda}$. Since $\varepsilon_{\lambda\delta} < 1$, this means that $\varepsilon_{v\delta}$ is always less than $\frac{1}{\nu}$.

Given the expression for the quit rate and for $\frac{\partial \ln q}{\partial \ln \lambda}$ in Equations (19) and (20),

$$\begin{aligned}\varepsilon_{q\delta} &= \frac{d \ln q}{d \ln \delta} = \frac{\partial \ln q}{\partial \ln \delta} + \frac{\partial \ln q}{\partial \ln \lambda} \frac{d \ln \lambda}{d \ln \delta} = 1 + \frac{\delta m_1 - \lambda s}{(\lambda s + \delta) m_1 - \lambda s} + \frac{\lambda s - \delta m_1}{(\lambda s + \delta) m_1 - \lambda s} \frac{d \ln \lambda}{d \ln \delta} = \\ &= 1 - \frac{\lambda s - \delta m_1}{(\lambda s + \delta) m_1 - \lambda s} (1 - \varepsilon_{\lambda\delta}).\end{aligned}$$

Given $\varepsilon_{\lambda\delta} < 1$ and $\frac{\lambda s - \delta m_1}{(\lambda s + \delta) m_1 - \lambda s} > 0$ as shown above, we get that $\varepsilon_{q\delta} < 1$.

8.9. Proof of Theorem 6. Again, the proof is algebraically very tedious, so here I only sketch its basic steps. The first step is to notice that, as a function of s and \bar{H} , h_λ can be written as

$$h_\lambda = \nu \frac{h_\lambda^1(s) - h_\lambda^2(s)r\bar{H}}{h_\lambda^3(s) - h_\lambda^4(s)r\bar{H}},$$

where the denominator is positive. Then, suppressing the dependence on s ,

$$\begin{aligned}\frac{\partial^2 h_\lambda}{\partial \bar{H} \partial s} &= \nu r \frac{[h_\lambda^4 h_\lambda^1 + h_\lambda^4 h_\lambda^{1'} - h_\lambda^{2'} h_\lambda^3 + h_\lambda^2 h_\lambda^{3'}] h_\lambda^3 - 2h_\lambda^{3'} h_\lambda^4 h_\lambda^2}{(h_\lambda^3 - h_\lambda^4 r \bar{H})^3} + \\ &+ \nu r \frac{h_\lambda^{4'} (h_\lambda^4 h_\lambda^1 - 2h_\lambda^2 h_\lambda^3) - [h_\lambda^4 h_\lambda^{1'} - h_\lambda^{2'} h_\lambda^3 - h_\lambda^2 h_\lambda^{3'}] h_\lambda^4}{(h_\lambda^3 - h_\lambda^4 r \bar{H})^3} r \bar{H}.\end{aligned}$$

To show that $\frac{\partial^2 h_\lambda}{\partial \bar{H} \partial s} < 0$ for a sufficiently large \bar{H} , it is enough to show that the numerator of the second term is negative. This can be done by showing that this expression is a quadratic function of ν that is increasing in ν on the interval $[0, 1]$, thus it is always smaller than its value at $\nu = 1$, which can be written as

$$h_\lambda^2 \frac{1}{s} \frac{y_1 - y_2}{y_1} \frac{y_2}{1 + y_2} \left(\frac{1}{1 + y_2} \left(\ln(1 + y_1) - \frac{y_1}{1 + y_1} \right) - \left(\frac{y_1}{1 + y_1} \right)^2 \right).$$

This expression is negative if the last term is negative, which happens if and only if

$$(1 + y_1)^2 (y_1 - \ln(1 + y_1)) > y_1^2 (y_1 - y_2).$$

This, in turn, holds if y_2 is sufficiently close to y_1 , which is the case if r is sufficiently small.

The statement regarding the cross-partial of $\varepsilon_{v\delta}$ follows from the fact that

$$\frac{\partial^2 \varepsilon_{v\delta}}{\partial s \partial \bar{H}} = \frac{\partial^2 \varepsilon_{\lambda\delta}}{\partial s \partial \bar{H}} \left(\frac{1}{\nu} - \frac{\lambda}{\delta + \lambda} \frac{(1 - s)\delta}{\delta + s\lambda} \right) + \frac{\partial \varepsilon_{\lambda\delta}}{\partial \bar{H}} \frac{\lambda\delta}{(\delta + s\lambda)^2}.$$

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FIGURES

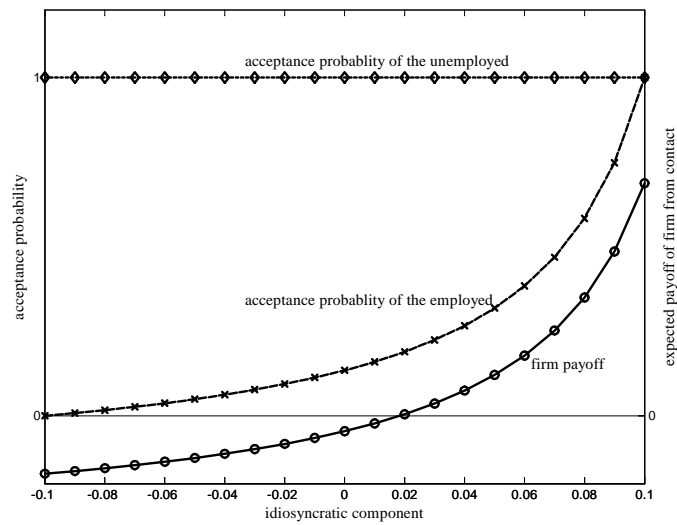


FIGURE 1. Firms' expected payoff from contacting workers with different idiosyncratic components and the probability of acceptance of unemployed and employed searchers as a function of their idiosyncratic component.

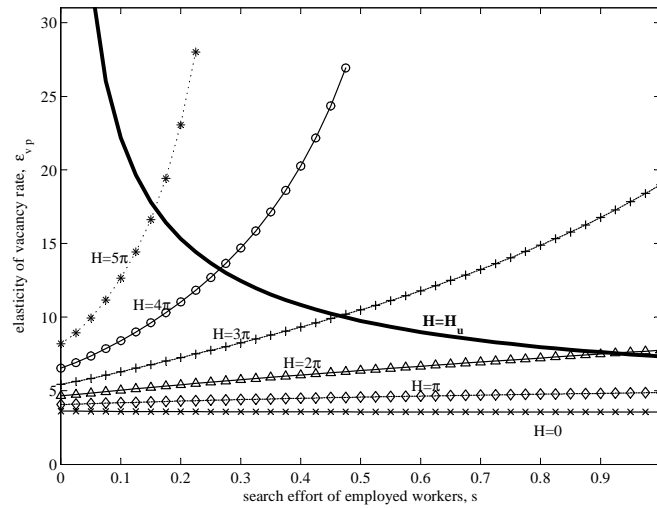
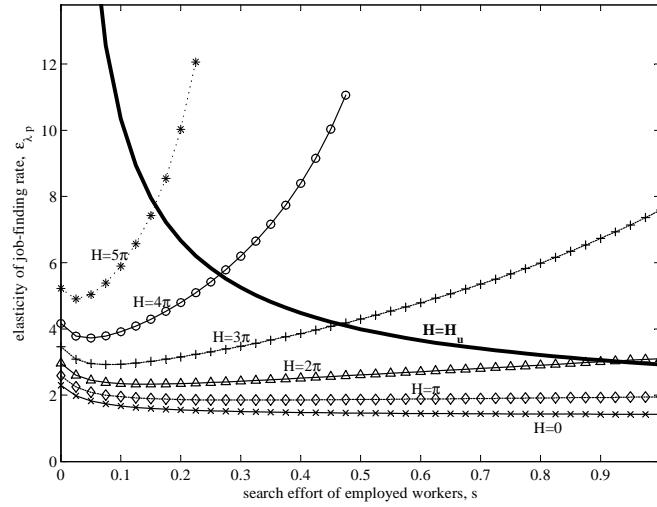


FIGURE 2. Elasticity of the job-finding rate (top panel) and of the vacancy rate (bottom panel) with respect to labor productivity as a function of the search effort of employed workers for different values of the hiring cost, H (expressed as a multiple of flow profits per quarter, π).

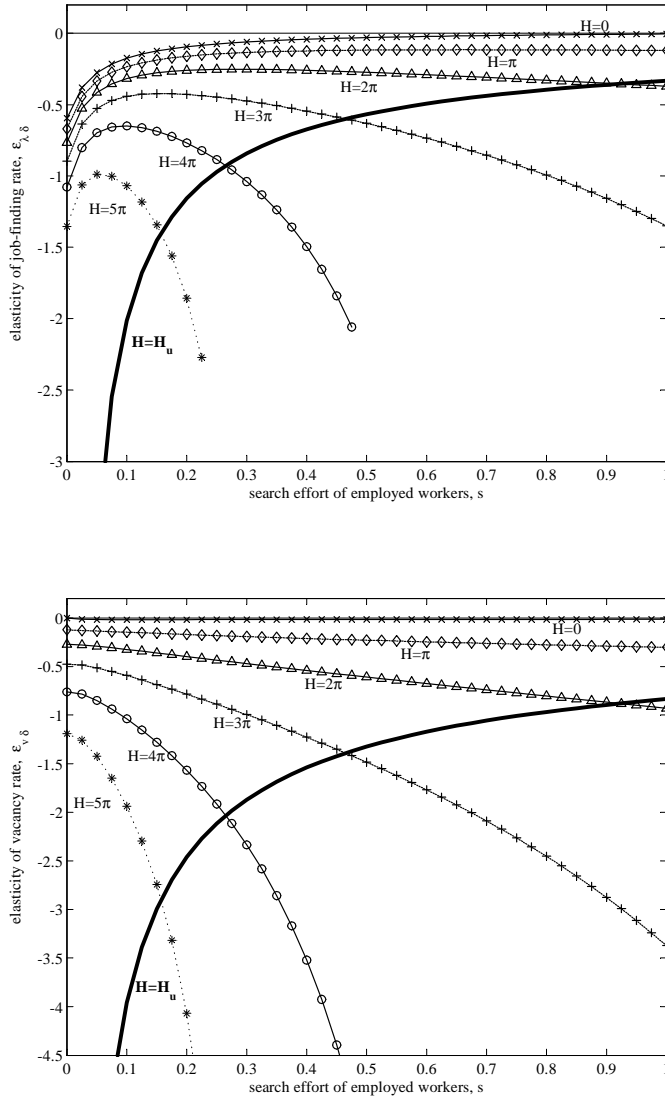


FIGURE 3. Elasticity of the job-finding rate (top panel) and of the vacancy rate (bottom panel) with respect to changes in the job-destruction rate as a function of the search effort of employed workers for different values of the hiring cost, H (expressed as a multiple of flow profits per quarter, π).

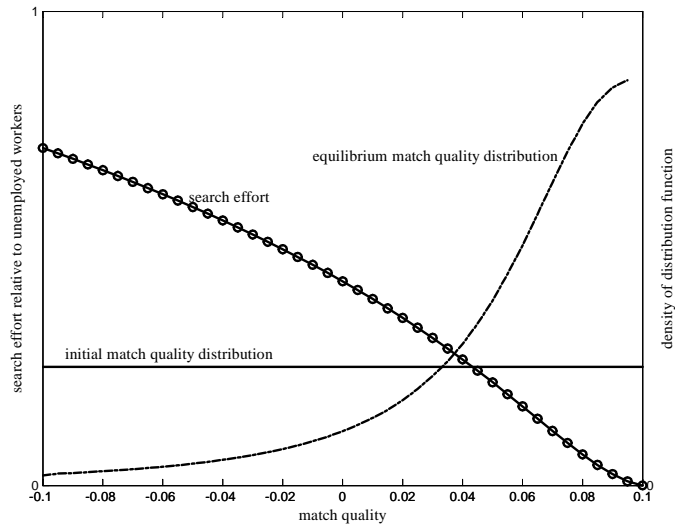
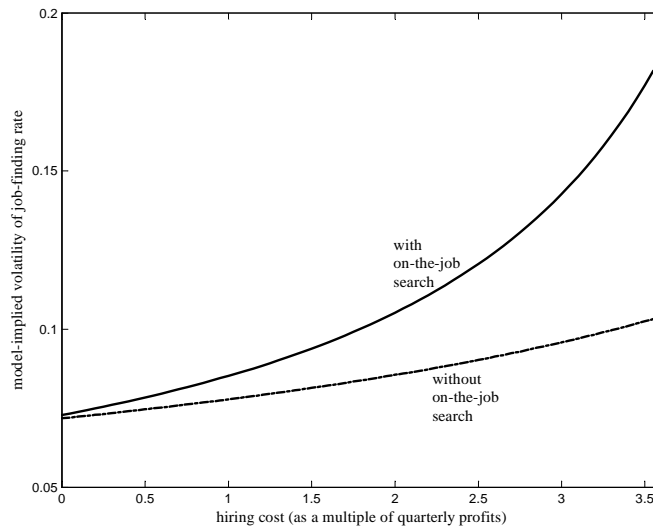


FIGURE 4. Equilibrium search effort as a function of the idiosyncratic component and the initial and equilibrium distribution of idiosyncratic component in the model with variable search effort.



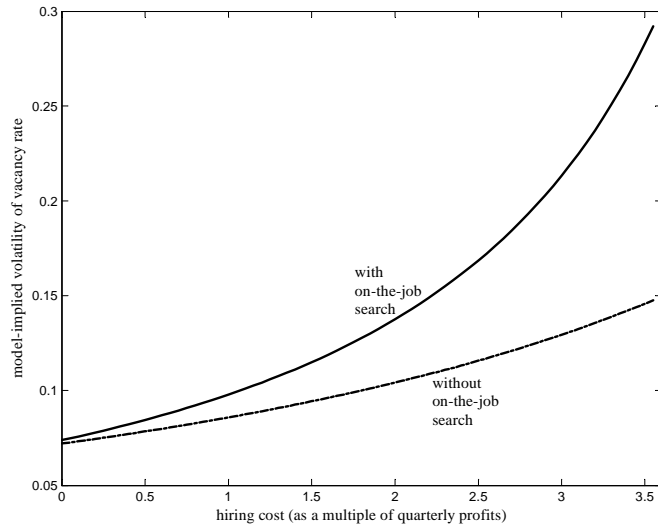


FIGURE 5. Model-implied volatility of the job-finding rate (top panel) and of the vacancy rate (bottom panel) as a function of the hiring cost with and without on-the-job search.