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On the Evaluation of Economic Mobility

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Abstract

This paper presents a framework for the evaluation and measurement of reversal and origin independence as separate aspects of economic mobility. We show how that evaluation depends on aversion to multi-period inequality, aversion to inter-temporal fluctuations, and aversion to future risk. We construct extended Atkinson indices that allow us to quantify the relative impact of reversal and origin independence on welfare. We apply our approach to the comparison of income mobility in Germany and in the United States. When aversion to inequality is the only consideration, the US gains more from mobility than Germany. This reflects similar gains from reversal in the two countries but greater gains in the US from origin independence. The introduction of aversion to intertemporal fluctuations and aversion to future risk makes the impact of mobility in the two countries more similar.

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1 Introduction

When is a society more 'mobile'than another? What are the welfare gains or losses (if any) associated with more or less mobility? It is widely recognized in the literature that these questions do not have simple answers. In a recent survey, Fields and Ok (2001) write:

> "...the mobility literature does not provide a unified discourse of analysis. This might be because the very notion of income mobility is not well-defined; different studies concentrate on different aspects of this multi-faceted concept. ... a considerable rate of confusion confronts a newcomer to the field."

In particular, the literature on mobility measurement has long recognized the tension between two different ways of measuring economic mobility: the degree to which ranks are 'reversed' over time and the degree to which future incomes do not depend on present income. In this paper, we will refer to those concepts, respectively, as 'reversal' and 'origin independence' (or, equivalently, 'time independence'). For example, in his important contribution to the axiomatic literature on mobility measurement, Shorrocks (1978a) argues that both principles should be maintained, since "interest in mobility is not only concerned with movement but also predictability - the extent to which future positions are dictated by the current place in the distribution." (Shorrocks, 1978a, p. 1016).

The literature on the measurement of mobility is mainly axiomatic, and, in general, does not provide explicit welfare foundations for the analysis of reversal and origin independence. Important exceptions are Atkinson (1981) and Atkinson and Bourguignon (1982). In their framework, welfare is maximized by complete reversal (all rich become poor and all poor become rich). Such approach is rooted in aversion to (multi-period) inequality, and captures an important dimension of the 'social value of mobility.' However, it leaves no role for origin independence. Some authors have seen this absence as at odds with the intuitive notion of mobility and with the idea that origin independence should have some value for society (Fields and Ok, 2001). By contrast, axiomatic (non-welfare-based) measures of mobility assign maximum 'mobility' to structures with perfect origin independence - e.g., Pais (1955) and Shorrocks (1978a).

In this paper, we propose a welfare framework that values both reversals *and* origin independence, and allows a separate evaluation of the welfare gains from each source. Our approach builds on the recognition that the welfare properties of mobility are closely linked to the theory of dynamic choice under uncertainty. The connection between choice under uncertainty and welfare analysis has long been explored in the literature on the measurement and evaluation of static inequality. For instance, in Atkinson's (1970) classic contribution inequality aversion is parametrized in ways formally equivalent to individual risk aversion. Harsanyi's (1955) 'veil-of-ignorance' concept has provided a philosophical link between individual choice under uncertainty and social choice. However, the relationship between choice under uncertainty and mobility is largely unexplored.[1]

One reason for such a gap is that mobility structures are irrelevant when welfare is evaluated using a time-separable Von-Neumann-Morgenstern (VNM) expected-utility framework, a formulation widely used in the analysis of intertemporal allocation. In that framework, the axiom of compound lotteries implies that only marginal distributions of outcomes have an impact on utility, while mobility patterns do not (unless they affect marginal distributions). In a standard expected-utility setting, two societies with very different degrees of mobility but identical marginal distributions must be evaluated identically.

We argue that a welfare foundation for mobility requires two steps. The first step is to move away from the standard time-additive expected-

[1]An interesting exception is Bénabou (2001), who studies the effects of progressive income taxes and redistributive finances on income, inequality, mobility, individual risk and intertemporal welfare. He does not focus on the measurement of mobility.

utility framework. The second step is to adopt a framework explicitly recognizing that mobility structures affects the 'predictability' of future outcomes. If second-period outcomes are not completely determined by first-period outcomes, removing the 'veil of ignorance' in the first period does not remove all 'uncertainty': an individual who knows her economic outcome today is still facing a 'lottery' in the following period. When the axiom of compound lotteries is abandoned, that dynamic pattern can matter for welfare. This allows time independence as well as reversal to affect the social value of mobility.

When Atkinson and Bourguignon's (1982) analysis is reinterpreted in terms of choice under uncertainty ('behind a veil of ignorance'), it becomes clear that they have taken the first step (abandoning time-additive expected utility) but not the second step (abandoning complete predictability). By contrast, in this paper we take both steps. We provide a welfare framework that is consistent with extensions of expected utility theory in which the axiom of compound lotteries does not hold.[2] In this framework the predictability of future outcomes matters. Specifically, we introduce preferences for the fundamentals that affect the social value of mobility: inequality, intertemporal fluctuations, and uncertainty. This allows us to construct indices that separate the welfare gains coming from reversal and time independence. Those indices can be used directly in empirical comparisons of mobility patterns across different societies.

In this paper, we use our approach in order to compare intragenerational mobility in Germany and in the United States, and we find the following:

a) When the focus of the welfare analysis is on multiperiod inequality, the effects of reversal are similar in Germany and in the United States, but

[2] As we will see, our analysis is related to the Kreps-Porteus (1978) axiomatization of choice under uncertainty in a dynamic setting. Applications of the Kreps-Porteus framework to consumption and saving decisions include Epstein and Zin (1989, 1991) and Weil (1990). More recently, the Kreps-Porteus axiomatization has been linked to the analysis of choice by "robust" decision makers - e.g., see Hansen, Sargent and Tallarini (1999).

the United States shows a much larger effect of time independence. This result suggests that mobility has similar effects on inequality reduction in the two countries after uncertainty is resolved, but that mobility provides higher utility behind a veil of ignorance in the US, since American income patterns are less 'predictable' than German ones.

b) American gains from less predictable patterns are accompanied by costs from larger economic fluctuations and more 'risky' income patterns. When aversion to income fluctuations and (beyond-the-veil) risk are introduced, those larger costs offset the benefits stemming from reduced multi-period inequality. Consequently, Germans and Americans end up obtaining similar net benefits from mobility, although for very different reasons.

The paper is organized as follows. Section 2 discusses the evaluation of mobility and introduces a social welfare framework that values both reversal and time independence. Section 3 presents our indices, which are built on the framework presented in previous section. In Section 4 we use our indices to compare mobility patterns in Germany and in the United States. Section 5 concludes.

2 The Value of Reversal and Time Independence

We start with a simple framework that highlights the analytical issues behind our approach (later we will show how the approach can be applied to more general classes of discrete and continuous distributions).

Consider a society in which individuals live for two periods.[3] In each period, half the population have low consumption (say $c^L > 0$) and the

[3] As usual in the literature, the analysis can be extended to the intergenerational case by reinterpreting 'individuals' as 'dynasties.' The welfare interpretation is basically unchanged if, because of altruistic links across generations, each dynasty can be viewed as an individual agent with a unique intertemporal utility function. In this paper examples

other half have high consumption (say $c^H > c^L > 0$). Let $\pi(c^i, c^j)$ denote the fraction of individuals who consume c^i ($i = L, H$) in the first period and c^j in the second period ($j = L, H$). Suppose that a fraction $(1 - \delta)$ of individuals have the same consumption level in both periods, while a fraction δ of individuals have different consumption levels:

$$\begin{bmatrix} \pi(c^L, c^L) = 1 - \delta & \pi(c^L, c^H) = \delta \\ \pi(c^H, c^L) = \delta & \pi(c^H, c^H) = 1 - \delta \end{bmatrix} \tag{1}$$

If the law of large numbers holds, the above fractions can be interpreted as probabilities, and the above matrix as a transition matrix.

This society will be called immobile if $\delta = 0$. Does 'mobility' ($\delta \neq 0$) have any value? And if it does, should one attach higher value to $\delta = 1/2$ (complete origin independence: second-period consumption is independent of first-period consumption) or to $\delta = 1$ (complete reversal: all the poor become rich and all the rich become poor)?[4]A natural starting point to address those questions is to consider a separable social welfare function W of the form

$$W = \sum_i \sum_j [u(c^i) + v(c^j)]\pi(c^i, c^j) \tag{2}$$

where $u(.)$ and $v(.)$ are concave (utility) functions. As long recognized in the literature (e.g., Markandya, 1982), if social welfare functions are time-separable and weigh utility from individual consumption levels according to their densities, only marginal distributions matter, and mobility has no

and applications will refer to intragenerational mobility. The two-period assumption is made for simplicity. An extension of the analysis to a multi-period setting is available upon request. Finally, while we focus on consumption, our approach can be extended to any utility-affecting variable. Because of data limitations, empirical analysis in the mobility literature often use income or earnings, even when consumption data would be theoretically preferable.

[4]As usual in this literature, we will evaluate different $\delta's$ taking the marginal distributions of consumption in each period as given. While our framework can also be used to evaluate *endogenous* links between mobility and marginal distributions, such analysis is not the focus of this paper.

welfare significance per se. In our example

$$W = \frac{1}{2}[u(c^L) + v(c^L) + u(c^H) + v(c^H)] \tag{3}$$

which does *not* depend on δ. In order to make mobility directly relevant from a welfare perspective, some intertemporal form of concavity must be introduced.[5]

Following Atkinson and Bourguignon (1982), consider a concave transformation of (1)

$$W = \sum_i \sum_j G[u(c^i) + v(c^j)]\pi(c^i, c^j) \tag{4}$$

where $G' > 0$ and $G'' < 0$. In our example

$$\begin{aligned} W &= \frac{1}{2}\{(1-\delta)G[u(c^L) + v(c^L)] + (1-\delta)G[u(c^H) + v(c^H)] \\ &\quad + \delta G[u(c^H) + v(c^L)] + \delta G[u(c^L) + v(c^H)]\} \end{aligned} \tag{5}$$

which implies $\frac{dW}{d\delta} > 0$ since G is concave,. Hence, any increase in δ improves social welfare, and the 'optimal' δ is equal to 1.

More generally, Atkinson and Bourguignon (1982) show that, for any social welfare function of the form $W = \sum_i \sum_j U(c^i, c^j)\pi(c^i, c^j)$ with $\frac{\partial U}{\partial c^i \partial c^j} < 0$, moving weight off the diagonal of a transition matrix is welfare improving. The resulting ranking of distributions is rooted in aversion to inequality. More precisely, the sign of $\frac{\partial U}{\partial c^i \partial c^j}$ depends on the relationship between aversion to inequality (which places positive value on reversal) and aversion to intertemporal fluctuations in consumption (which places negative value on reversal). If preferences are homothetic, the social welfare function used by Atkinson and Bourguignon is equivalent to

$$W = \{\sum_i \sum_j V^{1-\epsilon}\pi(c^i, c^j)\}^{\frac{1}{1-\epsilon}} \tag{6}$$

[5] An interesting alternative approach which maintains linearity but drops 'symmetry' (i.e., the assumption that each position receives equal weight in the social welfare function) has been developed by Dardanoni (1993).

where

$$V = (\alpha_1 (c^i)^{1-\rho} + \alpha_2 (c^j)^{1-\rho})^{\frac{1}{1-\rho}} \tag{7}$$

The parameter ϵ measures the degree of aversion to (multi-period) inequality, while ρ measures aversion to intertemporal fluctuations in consumption.[6]. $\dfrac{dW}{d\delta}$ is larger (equal, smaller) than 0 if and only if ϵ is larger (equal, smaller) than ρ. When $\epsilon > \rho$, the aversion to inequality offsets the aversion to intertemporal fluctuations, and the 'optimal δ' is equal to 1 (any increase in 'reversal' is welfare improving). Therefore, within the Atkinson-Bourguignon setting, if we prefer a mobile society ($\delta \neq 0$) to a static society ($\delta = 0$), we also prefer a society with complete reversal ($\delta = 1$) to any society with incomplete reversal ($\delta < 1$). The matrix with $\delta = 1/2$ (complete origin independence) has no special role in such a framework.[7]

How can the Atkinson-Bourguignon framework be extended in order to combine a valuation of both reversal and origin independence in a consistent way? Our proposal is to reinterpret the above analysis as a problem of dynamic choice under uncertainty. In particular, we use an approach that is consistent with a dynamic version of Harsanyi's (1955) veil-of-ignorance argument.

The connection between choice under uncertainty and welfare analysis has long been recognized in the literature on the measurement and evaluation of *static* inequality. For example, Atkinson's (1970) parametrization of social aversion to inequality is formally equivalent to that of individual risk aversion, and can be interpreted as reflecting aversion to risk behind a

[6]The parameters ϵ and ρ can take any nonnegative value except for 1. When $\epsilon = 1$, $W = \sum_i \sum_j (\ln V) \pi(c^i, c^j)$. When $\rho = 1$, $V = \alpha_1 \ln c^i + \alpha_2 \ln c^j$. The parameters α_1 and α_2 measure the relative weights in each period. In what follows, without much loss of generality, we will usually assume $\alpha_1 = \alpha_2 = 1/2$.

[7]An increase in reversal is associated with less time independence if and only if one restricts the analysis to matrices with positive dependence ($\delta < 1/2$ in our example) - e.g., see Conlisk (1990) and Bénabou and Ok (2000). However, such a restriction does not solve the conceptual issue of providing separate welfare-based evaluations and measures for the two different aspect of mobility. On this topic see Shorrocks (1978a).

veil of ignorance.[8] Under this interpretation, social welfare is given by the expected utility that a risk-averse individual would obtain if she were to put herself in the original position, in which the probability of each outcome were equal to the frequency of that outcome in the population.[9]

Analogously, the social welfare function proposed by Atkinson and Bourguignon (1982) can be interpreted as (multidimensional) expected utility.

More precisely, let E_0 be the expectation operator (the probability of each outcome being evaluated behind a veil of ignorance) defined over the joint distribution function of consumption levels.[10] Then, welfare can be written as

$$W = \{E_0[\alpha_1 (c_1)^{1-\rho} + \alpha_2 (c_2)^{1-\rho}]^{\frac{1-\epsilon}{1-\rho}}\}^{\frac{1}{1-\epsilon}} \tag{8}$$

The above expression contains an implicit assumption: when the veil of ignorance is removed and the identity of each individual is known, *all* uncertainty is removed, and each individual's consumption path is also known with certainty. But uncertainty about period-2 consumption for given period-1 consumption is at the core of origin independence: only in a society with complete immobility ($\delta = 0$) or complete reversal ($\delta = 1$),

[8]In fact, Atkinson's interest in the question of measuring inequality was originally stimulated by an early version of Rotschild and Stiglitz's (1970) fundamental contribution to the literature of decision-making under uncertainty. However, in his 1970 paper Atkinson did not explore the philosophical connection between individual decision under uncertainty and social choice, but chose to view the two problems as "formally similar" but "economically unrelated" (Atkinson, 1970, p. 245).

[9]The law of large numbers needs to hold for the individual's utility function to be reinterpreted as a social welfare function. In this paper we will assume that such condition holds. On this important topic see Judd (1985).

[10]Formally, let $f(c_1, c_2)$ be the joint distribution function, with $c_1 \in C_1$ and $c_2 \in C_2$, where C_1 and C_2 are either discrete or continuous sets. Let $\psi(c_1, c_2)$ be any function of c_1 and c_2.Then, $E_0[\psi(c_1, c_2)] \equiv \sum_{C_1} \sum_{C_2} \psi(c_1, c_2) f(c_1, c_2)$ if C_1 and C_2 are discrete sets, and $E_0[\psi(c_1, c_2)] \equiv \int_{C_1} \int_{C_2} \psi(c_1, c_2) f(c_1, c_2) dc_1 dc_2$ if C_1 and C_2 are continuous sets. In our example with only two states $E_0[\psi(c_1, c_2)] = \psi(c^L, c^L)\pi(c^L, c^L) + \psi(c^L, c^H)\pi(c^L, c^H) + +\psi(c^H, c^L)\pi(c^H, c^L) + \psi(c^H, c^H)\pi(c^H, c^H)$.

If either α_1 or α_2 is zero, W reduces to a standard static expected utility behind a veil of ignorance, and ε is Atkinson's (1970) aversion to inequality.

knowing an individual's consumption in period 1 would be sufficient to predict that individual's consumption in period 2. By implicitly assuming away those dynamic aspects of uncertainty, the Atkinson-Bourguignon approach cannot attribute a role to time independence.

We propose to relax the extreme assumption that consumption paths become known with certainty once the veil of ignorance about individuals' identities is removed. Instead, we assume that, in period 1, individuals do not know period 2 outcomes with certainty, but take *conditional expectations of* c_2 *based on their observed* c_1 *and the joint density of outcomes.*

Specifically, we extend the Atkinson-Bourguignon framework by considering certainty-equivalent values of consumption in period 2. Maintaining the isoelastic specification, we introduce a new parameter γ, which measures aversion to second-period risk. The existence of second-period risk can be viewed as stemming from a dynamic extension of the veil of ignorance argument.[11] That is, the veil of ignorance is only partially removed in period 1 (when individuals know their consumption levels in period 1), but it is maintained with respect to period 2, conditionally on consumption in period 1. The parameters ϵ and γ are closely related (they both measure aversion to some risk), but they are conceptually and ethically distinct: ϵ measures aversion to multi-period inequality, while γ measures aversion to risk in second-period consumption, once first-period consumption is known.

The certainty equivalent of second-period consumption is given by[12]

$$\widehat{c}_2 = \{E_1[c_2^{1-\gamma}|\ c_1]\}^{\frac{1}{1-\gamma}} \tag{9}$$

where E_1 is the mathematical expectation conditional on information available in period one, which includes both first-period consumption levels and the joint density of outcomes.[13] By substituting second-period consumption

[11] We thank an anonymous referee for suggesting this point.

[12] As usual, γ can take any nonnegative real value, except for 1, in which case we have $\widehat{c}_2 = E_1[\ln c_2|\ c_1]$.

[13] Formally, for any function $\varphi(c_2)$, we define $E_1[\varphi(c_2)|\ c_1]$ as $E_1[\varphi(c_2)|\ c_1] \equiv$

with its certainty-equivalent in equation (8), and assuming for simplicity $\alpha_1 = \alpha_2 = 1/2$, the social welfare function becomes:

$$\widehat{W} = \{E_0[\frac{1}{2}{c_1}^{1-\rho} + \frac{1}{2}{\widehat{c}_2}^{1-\rho}]^{\frac{1-\epsilon}{1-\rho}}\}^{\frac{1}{1-\epsilon}} \tag{10}$$

If $\epsilon = \rho = \gamma$, the social welfare function $\widehat{W}$ reduces to a standard, additively separable isoelastic VNM utility function. When the three parameters are not identical, the social welfare function is consistent with a more general class of preferences, for which the axiom of compound lotteries is not necessarily satisfied. Specifically, Kreps and Porteus (1978) provide an axiomatic foundation of preferences when 1) the axiom of compound lotteries is abandoned; 2) all other axioms of VNM utility theory are maintained; 3) the temporal consistency of optimal plans is imposed axiomatically.

Specifically, Kreps-Porteus preferences link attitudes towards temporal resolution of uncertainty with attitudes toward risk aversion (aversion to fluctuations of consumption across 'states') and intertemporal substitution (aversion to fluctuations of consumption across 'dates'). A heuristic explanation of the relationship between aversion to intertemporal fluctuations, aversion to risk, and preferences for the timing of the resolution of uncertainty has been provided by Philippe Weil (1990). Weil notes that lotteries in which uncertainty is resolved earlier are 'less risky' ('safer') than later-resolution lotteries with the same distribution of prizes. However, early resolution implies larger fluctuations of utility over time (later-resolution lotteries are 'more stable'). There is a trade off between 'safety' and 'stability' of utility. Agents who dislike intertemporal fluctuations more (less) than risk will tend to prefer late (early) resolution. Epstein and Zin (1991) estimate the parameters that determine the attitudes toward risk and intertemporal substitution for a time-invariant isoelastic representation of

$\sum_{C_2} \varphi(c_2) f(c_1, c_2)$ if C_2 is a discrete set, and $E_1[\varphi(c_2)|\ c_1] \equiv \int_{C_2} \varphi(c_2) f(c_1, c_2) dc_2$ if C_2 is a continuous set.

Kreps-Porteus preferences, and find moderate degrees of risk aversion (a coefficients of relative risk aversion γ around 1) but larger aversion to intertemporal fluctuations. A more recent line of research (Hansen, Sargent and Tallarini, 1999) estabilishes a relationship between the Kreps-Porteus axiomatization and the representation of preferences when agents are 'robust' decision makers - that is, when agents suspect specification errors and want decisions to be insensitive to them. The robustness approach is also closely related to the max-min utility theory of Gilboa and Schmeidler (1989) and Epstein and Wang (1994), and therefore indirectly links the Kreps-Porteus axiomatization to those other extensions of expected utility theory.

Our framework can be viewed as an isoelastic application of the Kreps-Porteus framework to encompass evaluations behind-a-veil-of-ignorance.[14] As in the Atkinson-Bourguignon specification (which our broader framework encompasses), the social welfare function values reversals if and only if $\epsilon > \rho$. However, unlike in Atkinson-Bourguignon, it is now possible to determine a range of parameters such that social value is also given to time independence. Preferences for the timing of uncertainty resolution depend not only on parameters ρ and γ (as in standard Kreps-Porteus isoelastic specifications), but also on ε. The following proposition shows what restrictions on the preference parameters are required for time independence to be valued - i.e., for welfare under (10) to be larger than welfare under (8):

Proposition 1: *Time independence is positively valued if and only if* $\max\{\varepsilon, \rho\} > \gamma$ and $\min\{\varepsilon, \rho\} \geq \gamma$. *That is, time independence is valued if* $\varepsilon \geq \gamma$ *and* $\rho \geq \gamma$, *and at least one inequality is strict.*

Proof: Appendix A.1

[14] In Appendix A.3 we present an extension of our welfare function that is consistent with a more general specification of Kreps-Porteus preferences.

Intuitively, higher aversion to risk in the second period (γ) implies a higher cost from unpredictability of second-period consumption. For time independence to have value, the other two parameters must be 'high enough' to compensate for that cost.

A positive evaluation of time independence does not necessarily mean that *complete* time independence is socially optimal. Going back to our 2x2 example, full time independence ($\delta = 1/2$) is optimal only if the social welfare function assigns no weight to reversals ($\epsilon = \rho$). If individuals care about reversals ($\varepsilon > \rho$) but also about time independence, they face a trade-off between the two goals: a δ closer to 1 gives more reversal, while a δ closer to 1/2 reduces predictability, and the 'optimal' δ lies between 1/2 and 1. The converse is true for $\varepsilon < \rho$. Formally we have the following

Proposition 2: *When the conditions in Proposition 1 are satisfied*, *the value of* δ *that maximizes welfare, as in equation (11), is larger/equal/smaller than 1/2 if* ε *is larger/equal/smaller than* ρ.

Proof: Appendix A.2

This result, which the above Proposition 2 illustrates for the simplest possible case (a discrete bivariate distribution with two points of support), extends to more general distributions. A generalization for continuous distributions with linear projection of second-period consumption is available from the authors.

As shown in Appendix A.3, the analysis can be generalized to a larger family of social welfare functions. In the rest of this paper we will focus on the isoelastic case. That specification not only allows us to parametrize preferences by using ϵ, ρ, and γ, each related to a different 'fundamental' (aversion to multiperiod inequality, fluctuations across periods, and second-period risk), but is a natural foundation for a family of indices that we will present in the following section and use in our empirical analysis.

3 Evaluating Reversal and Time Independence: Extended Atkinson Indices

In this section we use our framework to construct welfare-based indices based on the preference parameters ε, ρ and γ. Those indices allow us to quantify the welfare value of reversal and time independence and provide comparisons across different societies.

Our starting point is the level of welfare that a society would have achieved in the *absence* of mobility. Let W_s denote the level of welfare obtained in a completely immobile society - i.e., for each individual i, substitute her actual c_2^i with c_{12}^i, which denotes the level of second-period consumption that individual would have obtained if she had maintained her first-period rank.[15] Hence

$$W_s = \{E_0[\frac{1}{2}(c_1^i)^{1-\rho} + \frac{1}{2}(c_{12}^i)^{1-\rho}]^{\frac{1-\epsilon}{1-\rho}}\}^{\frac{1}{1-\epsilon}} \tag{11}$$

By construction, such a static society has no reversal and no origin independence. As long as individuals do not like inequality ($\varepsilon \geq 0$) and/or intertemporal fluctuations ($\rho \geq 0$), they would prefer a society in which, in each period, everybody receives the average level of (multi-period) consumption

$$\bar{c} \equiv E_0 \frac{c_1 + c_2}{2} \tag{12}$$

to the static society ($W_s \leq \bar{c}$). The gap between $\bar{c}$ and W_s measures how much the static society would gain if inequality of consumption (across individuals and across periods) were eliminated. Following Atkinson (1970), that gap can be reinterpreted in light of the following question: What is

[15] The idea of using a hypothetical benchmark structure in which individual ranks are maintained is well estabilished in the literature on mobility indices. For instance, King (1983) uses such a benchmark in order to obtain an index that measures changes in the rank orders of the income distribution. See also Chakravarty (1984) and Chakravarty, Dutat and Weymark (1985).

the fraction of $\overline{c}$ that the static society would be willing to sacrifice in order to achieve a fully egalitarian distribution of consumption across individuals and across periods?

The following 'extended' Atkinson index provides the answer

$$A_s = 1 - \frac{W_s}{\overline{c}} \tag{13}$$

A_s is a measure of relative welfare loss from inequality.[16] Its close relation to the standard Atkinson inequality index becomes fully apparent when the marginal distributions in the two periods are identical (i.e., when $c_{12}^i = c_1^i$ for every i). In that case, W_s reduces to

$$W_s = \{E_0 c_1^{1-\epsilon}\}^{\frac{1}{1-\epsilon}} = \{E_0 c_2^{1-\epsilon}\}^{\frac{1}{1-\epsilon}} \tag{14}$$

and A_s coincides with the standard Atkinson's inequality index for the marginal distribution.

Now, consider how welfare is improved through reversal (but, for the moment, without introducing origin independence). Let W_r denote welfare in a society in which individuals enjoy their actual levels of consumption in period 2, and second-period consumption is known with certainty in period 1 ($E_1[c_2 \mid c_1] = c_2$). Then

$$W_r = \{E_0[\frac{1}{2}{c_1}^{1-\rho} + \frac{1}{2}{c_2}^{1-\rho}]^{\frac{1-\epsilon}{1-\rho}}\}^{\frac{1}{1-\epsilon}} \tag{15}$$

[16] All extended Atkinson indices presented in this paper are relative indices (they remain unchanged when consumption levels are scaled proportionally). More generally, the literature on cooperative decision making has identified a set of axioms (separability, independence of common scale, inequality reduction) that are uniquely satisfied by the isoelastic family of social welfare functions, defined over individual utility levels (see Roberts, 1980 and Moulin, 1988, chapter 2). Those results apply to our indices insofar as one interprets $u_i = [\frac{1}{2}{c_1}^{1-\rho} + \frac{1}{2}(E_1 c_2^{1-\gamma})^{\frac{1-\rho}{1-\gamma}}]^{\frac{1-\epsilon}{1-\rho}}$ as individual i's utility level. While this suggests a possible avenue to provide our indices with an axiomatic foundation, such analysis is beyond the scope of this paper.

As long as individuals dislike inequality ($\varepsilon \geq 0$) and/or fluctuations ($\rho \geq 0$), $W_r \leq \bar{c}$. Analogously to the static-society case, we can build an extended Atkinson index for this 'predetermined' society

$$A_r = 1 - \frac{W_r}{\bar{c}} \tag{16}$$

A_r measures the fraction of consumption $\bar{c}$ that individuals in a society with reversal (but complete knowledge about c_2) would be willing to sacrifice in order to achieve equality of consumption. If reversal increases welfare ($\varepsilon > \rho$) and $c_2^i \neq c_{12}^i$ for some i, we have that $W_r > W_s$, which implies $A_r < A_s$ The difference $A_s - A_r$ measures the *reduction* (caused by reversal) in the fraction of consumption society would be willing to sacrifice in order to eliminate inequality and fluctuations.

The impact of origin independence can be captured similarly. $\widehat{W}$, defined in equation (10), measures welfare taking into account the actual degree in which second-period consumption depends on first-period consumption. Again, we can build an extended Atkinson index

$$A_o = 1 - \frac{\widehat{W}}{\bar{c}} \tag{17}$$

A_o measures the fraction of consumption $\bar{c}$ that individuals are willing to sacrifice in order to achieve equality of consumption across people and across periods. By comparing A_r and A_o, we can assess the *welfare impact of origin dependence*. As long as origin independence is valued (i.e., $max\{\varepsilon, \rho\} > \gamma$ and $min\{\varepsilon, \rho\} \geq \gamma$), imperfect predictability increases welfare and, therefore, reduces A_o with respect to A_r. Hence, the difference between $A_r - A_o$ measures the welfare impact of time independence.

The overall impact of mobility can be evaluated by decomposing the difference between A_s and A_o into its two components

$$A_s - A_o \equiv (A_s - A_r) - (A_r - A_o) \tag{18}$$

In the following section we use this framework to compare mobility in the United States and Germany.

4 Empirical Application: A Comparison of the United States and Germany

In this section we apply our measures to study differences in intragenerational family income mobility in the United States and Germany. While our primary focus is on illustrating the use of our indices, our application also makes a substantive contribution to the literature on cross-national comparisons of mobility . Studies of intragenerational mobility include Aaberge,et al (2000), who compare family income mobility in the US. with several Nordic countries, and Burkhauser et al. (1998), who compare family income mobility in the US. and Germany. OECD (1997) also presents a variety of comparisons across OECD countries.[17] All these studies use standard measures of mobility, such as differences in transition matrices, differences in regression or correlation coefficients, or differences in the reduction in inequality from extending the accounting period - as suggested in the theoretical contribution by Shorrocks (1978b). In his analysis of US and Italian data, Flinn (2000) compares inequality of cross-sectional wage distributions and distributions of lifetime welfare. The later are estimated from a search-theoretical model of optimal job transitions.

The best evidence on mobility in Germany and the US comes from Burkhauser et al. (1998) who find that the diagonal elements of the German quintile transition matrices of post-government family income are somewhat smaller in Germany than in the US. While their study provides an important starting point, the measure they use does not have an explicit welfare

[17] See Björklund and Jäntti (2000) for a review of the comparative literature on intergenerational mobility. This literature also relies on standard measures that do not separate the effects of reversal from time independence.

interpretation, and cannot provide an evaluation of the relative importance of reversal and origin independence in the two countries. In this section we compare these two different aspects of mobility in the two countries under explicit assumptions about the values placed on multi-period equality (ε), intertemporal stability (ρ) and risk (γ).

Our data, like Burkhauser et al.'s (1998), come from the equivalent files of the German Socio-Economic Panel (SEOP) and the Panel Income of Income Dynamics (PSID). These two data sets offer the advantage of having similar design and similar definitions for the key variables needed for this study . Both data sets also cover a sufficiently long period to capture permanent changes in incomes.[18] We use data for 1984 and 1993, which is the longest period over which we have consistent data for both countries. 1984 is the earliest year of data for Germany and 1993 is the latest year of final release data for the PSID.[19] Our data cover all persons 25 to 55 in 1984. Persons with zero sampling weights are excluded since our measures are calculated using sample weights designed to make the samples nationally representative. The German sample also excludes the East German sample since this sample was only added after 1984.

Our measure of income is post-tax and transfer family income, adjusted for family needs using the US equivalence scales. The top and bottom one percent of the marginal distributions are trimmed in each year in order to eliminate outliers. Our measures of mobility based on trimmed data can, therefore, be interpreted as movement within the interior 98 percentile of the joint distribution in each country[20].

[18]As is standard practice in the mobility literature, we are limited to using data on income, since data on consumption is not available.

[19]Early release data are not wholly consistent with the final release data that we use. For example there is a sharp change in the proportion of persons living in households with zero family income.

[20]While our sample differs from Burkhauser et al. in minor ways, our data give similar results using their measures. The 3.8 percentage point difference in the probability of staying in the same quintile is very close to the difference of 3.6 percentage points they find.

Before turning to our measures, we present the basic information on the joint distributions of income that is summarized by any measure of mobility.[21] Figure 1 shows the kernel smoothed contours of the joint distributions of the log of 1984 and 1993 income for the US and Germany. In order to center both distributions we shows the log deviations from each country's mean.[22] Contours are drawn at the densities that separate the 20th, 40th, 60th, and 80th percentiles. These simple plots immediately illustrate three key differences between the two countries. First, the contours for Germany lie wholly within those of the US. This shows the remarkable degree to which Germany has a more equal cross-sectional distribution than the US. Second, since income movement is measured by the vertical distance from the 45 degree line, the US would seem to offer greater income changes. Third, the contours for Germany are somewhat flatter than for the US. This indicates that the expected value of 1993 income increases less with 1984 income in Germany than in the US. As a result, standard measures based on regression coefficients or correlations in income across time would show Germany having more mobility than the US since the conditional mean of 1993 varies less with 1984 income in Germany than in the US. However, the dispersion around these conditional means is greater in the US than in Germany. The latter indicates that there is greater uncertainty around the conditional mean. In terms of our analysis, this suggests a larger role for time independence in the US than in Germany.

We now turn to our measures of mobility that are based on explicit values for the underlying parameters that determine the value of reversal and time independence. Tables 1 shows values for the three extended Atkinson indices we have derived in Section 4. Since the values of each of our indices depend on values of the underlying preference parameters, each column is

[21] Transition matricies are discretized versions of these joint distributions. Appendix A.4 presents these matricies for comparison to other studies.

[22] We thank Markus Jantti for graciously providing plots of the joint density.

calculated for different values of ε, ρ, and γ.[23] Column 1 assumes that there is a preference for equality ($\varepsilon = 4$) but no aversion to intertemporal fluctuations or to second period risk ($\rho = \gamma = 0$). These values are chosen to illustrate the basic links between aversion to multiperiod inequality and the value of reversal and time independence. While we will also calculate our indices for nonzero measures of ρ and γ, setting those values to zero provides a useful benchmark.

Column 1 shows that when ε is equal to 4 and ρ and γ are both zero, A_s is equal to .666 for US and .406 for Germany. The fact that the extended Atkinson index for a static society is substantially lower in Germany than in the US indicates that the marginal distributions are considerably more equal in Germany than in the US, which is consistent with the plots in Figure 1.

The values of A_r in row 2 show the extended Atkinson index after allowing for reversal. Allowing persons to change places in the marginal distributions lowers the extended Atkinson index by .101 in the US and by .117 in Germany. Given an inequality aversion parameter of 4, both countries would be willing to give up around ten percent of multi-period income in order to maintain their level of reversal. This indicates that reversal has a similar impact in raising welfare in the US and Germany.[24]

Row 3 of Table 1 shows , A_o. This index captures the gains behind the veil of ignorance from not knowing second period income with certainty. The difference between A_o and A_r therefore shows the gains from time independence. In this case, the US exhibits substantially larger gains from

[23]The displayed values satisfy the conditions for positive value for reversal (as $\varepsilon > \rho$) and for time independence (since $min\{\varepsilon, \rho\} > \gamma$). Indices for the full array of parameters are available from the authors.

[24]Since our measures themselves are indices, they show the percentage point increase in well-being (measured as a fraction of equivalent income) from reversal and time independendence. If one calculated the *percentage change* in the index, Germany would experience a larger percentage decline, since it starts from a lower base. We see no rational for doing this.

mobility than Germany. For the US time independence has a value of .211. In contrast the value of .115 for Germany is roughly half as large. The higher value of time independence in the US is again consistent with Figure 1, which indicates greater dispersion around the conditional mean. The bottom row of column 1 gives the net impact of reversal and time independence when there is no aversion to intertemporal fluctuations or risk. Here we see that the US benefits more from mobility. As we have seen, this reflects differences in time independence not reversal, which is similar in the two countries.

In summary, the main message from column 1 is that, when one focuses on inequality reduction (which is the focus of most applied literature on mobility), the effects of reversal on welfare are similar in the US and in Germany, but the US shows much larger gains from time independence. This is consistent with the fact that, overall, income seems to be less predetermined in the US than in Germany.[25]

When nonzero values for ρ and γ are introduced, the costs from intertemporal fluctuations and second-period risk partially offset the benefits from inequality reduction associated with mobility. Those negative effects are larger in the US than in Germany. In particular, when individuals value fluctuations and risk negatively, the US sees a sharper reduction in its net benefits from time independence. As a consequence, the net benefits of mobility for the two countries narrow. Column 2 introduces aversion to intertemporal fluctuations by setting ρ equal to 2 . When ρ is raised from 0 to 2 the gains to reversal are cut roughly in half in both the US. and Ger-

[25] Our findings may shed some light on the paradox that, while standard reversal-based comparisons of mobility patterns between the US and Germany (or other European countries) present similar measures or are inconclusive, observers often seem to perceive a higher degree of mobility in the US than in Europe. For example, Alesina, Di Tella and MacCulloch (2000) document the different evaluation of inequality by Europeans and Americans and relate it to differences in perceived social mobility. Our analysis suggests that the US may have higher perceived mobility than Germany insofar as mobility is measured by the welfare effects of *time independence.*

many. Since the two countries started with similar gains from reversal, the introduction of aversion to intertemporal fluctuations, leaves both countries with smaller but similar gains(.046 for the US and .055 for Germany). The effects on the gains from origin independence are, however, very different. As we saw in column 1 Germany gains relatively little from origin independence when there is no aversion to temporal fluctuations. Raising ρ from 0 to 2 has relatively little impact on the values for Germany, lowering the value from .115 to .101. In contrast the value of origin independence is cut nearly in half in the US, from .211 to .114. As a result, when ε is equal to 4 and ρ is equal to 2, Germany and the US gain roughly equally from origin independence.

Column 3 introduces aversion to 1993 risk by setting γ equal to 2. Since both A_s and A_r are based on the actual realizations of 1993 income, the value of reversal, which is measured by the difference between these two Atkinson indices, is unaffected by this parameter. Introducing aversion to risk, however, reduces the value of time independence, since a risk premium must now be paid for the variation of realizations of 1993 income around its expectation. Again, the value of time independence is cut roughly in half for both countries. But since the values of time independence in column 2 are roughly the same in the US and Germany, cutting both in half leaves the two countries with similar gains.

In summary, we have shown that if aversion to inequality is the only consideration, then the US gains more from mobility than Germany. This reflects greater gains in the US from origin independence but similar gains from reversal. If, in addition there is aversion to intertemporal fluctuations or risk (i.e. γ or ρ are not equal to zero), then the US and Germany have similar overall gains from mobility. These overall gains reflect roughly equal gains from reversal and time independence. More generally, we have found that, as ρ and/or γ increase, the impact of mobility (in its two aspects) tend to improve in Germany relatively to the US.

5 Conclusions

We have provided a general framework that allows us to separate the value of mobility as reversal from the value of mobility as origin independence. In particular, we have provided an isoelastic social welfare function that links the evaluation of those two aspects of mobility to preferences for fundamentals: aversion to multi-period inequality (parametrized by ε), aversion to intertemporal fluctuations (parametrized by ρ), and aversion to future risk (parametrized by γ). Reversal reduces multi-period inequality but increases intertemporal fluctuations. Consequently, individuals positively evaluate reversal when aversion to inequality dominates aversion to intertemporal fluctuations (ε is larger than ρ). Origin independence reduces both multi-period inequality and intertemporal fluctuations, but increases future risk. Individuals will positively value origin independence as long as aversion to multi-period inequality and aversion to fluctuations dominate aversion to future risk (ε and ρ are not smaller than γ, and at least one of them is larger).

Using our framework, we have introduced extended Atkinson indices that answer the following question: What fraction of its average consumption would a society be willing to sacrifice in order to eliminate multi-period inequality, intertemporal fluctuations and future risk? We have provided extended Atkinson indices under complete immobility (A_s), under fully predictable reversal (A_r), and under the observed degrees of reversal and origin independence (A_o). The difference between A_s and A_r is a measure of the welfare gains from reversal, while the difference between A_r and A_o measures the welfare gains from origin independence. The overall gains from mobility are given by the sum of the gains from reversal and origin independence.

By applying this approach to the comparison of intragenerational mobility patterns in Germany and in the US, we have found some intriguing cross-national differences in the relative impact of reversal and origin independence. When aversion to inequality is the only consideration (i.e. $\rho = \gamma = 0$), the US gains more from mobility than Germany. This reflects similar gains from reversal in the two countries but greater gains in the US from origin independence. The introduction of aversion to intertemporal fluctuations and aversion to second-period risk makes the impact of mobility in the two countries more similar, with both gaining about equally from reversal and origin independence.

APPENDIX

A.1. Derivation of Proposition 1

By definition, later resolution of uncertainty is preferred for all marginal distributions of consumption if and only if the following holds for all nondegenerate distributions of c_1 and c_2, (where c_1 and c_2 are strictly positive):

$$\left\{E_0\left[(1-\beta)c_1^{1-\rho}+\beta[E_1[c_2^{1-\gamma}|c_1]^{\frac{1-\rho}{1-\gamma}}\right]^{\frac{1-\varepsilon}{1-\gamma}}\right\}^{\frac{1}{1-\varepsilon}}$$
$$> \left\{E_0\left[(1-\beta)c_1^{1-\rho}+\beta(c_2^{1-\gamma})^{\frac{1-\rho}{1-\gamma}}\right]^{\frac{1-\varepsilon}{1-\gamma}}\right\}^{\frac{1}{1-\varepsilon}} \tag{19}$$

Define

$$G(x)\equiv\left[(1-\beta)c_1^{1-\rho}+\beta x^{\frac{1-\rho}{1-\gamma}}\right]^{\frac{1-\varepsilon}{1-\gamma}} \tag{20}$$

where $x\equiv c_2^{1-\gamma}$. The above inequality (19) holds if and only if

i) for $\varepsilon<1$, we have that

$$G(E_1x)>E_1G(x) \tag{21}$$

for all distributions of c_1 and c_2, that is, if $G(x)$ is concave in x (Jensen's inequality).

ii) for $\varepsilon>1$, we have that

$$G(E_1x)<E_1G(x) \tag{22}$$

for all distributions of c_1 and c_2, that is, if $G(x)$ is convex in x (Jensen's inequality).

The conditions under which (21) and (22) hold can be derived by defining

$$m\equiv(1-\beta)c_1^{1-\rho} \tag{23}$$

$$n \equiv \beta x^{\frac{1-\rho}{1-\gamma}} \tag{24}$$

$$p \equiv \frac{\beta x^{\frac{1-\rho}{1-\gamma}-2}\left[(1-\beta)c_1^{1-\rho}+\beta x^{\frac{1-\rho}{1-\gamma}}\right]^{\frac{1-\varepsilon}{1-\gamma}-2}}{(1-\gamma)^2} \tag{25}$$

Note that m, n and p are all strictly positive for positive values of c_1 and c_2.

As

$$G''(x) = (1-\varepsilon)p[(\gamma-\rho)m+(\gamma-\varepsilon)n] \tag{26}$$

we have that

i) for $\varepsilon < 1$, $G''(x) < 0$ for all positive values of m and n if and only if $\gamma \leq \rho$ and $\gamma \leq \varepsilon$ (with at least one inequality being strict).

ii) for $\varepsilon > 1$, $G''(x) > 0$ for all positive values of m and n if and only if $\gamma \leq \rho$ and $\gamma \leq \varepsilon$ (with at least one inequality being strict).

QED

A.2 Derivation of Proposition 2

The first derivative of (3.6) with respect to δ can be written as follows

$$W'(\delta) = Q(\delta)S(\delta) \tag{27}$$

where

$$Q(\delta) \equiv \frac{c_H^{1-\gamma} - c_L^{1-\gamma}}{4(1-\gamma)} W^{\varepsilon} \tag{28}$$

and

$$\begin{aligned} S(\delta) \quad \equiv \quad & \{\frac{1}{2}c_L^{1-\rho} + \frac{1}{2}[(1-\delta)c_L^{1-\gamma} + \delta c_H^{1-\gamma}]^{\frac{1-\rho}{1-\gamma}}\}^{\frac{\rho-\varepsilon}{1-\rho}}[(1-\delta)c_L^{1-\gamma} + \delta c_H^{1-\gamma}]^{\frac{\gamma-\rho}{1-\gamma}} + \\ & -\{\frac{1}{2}c_H^{1-\rho} + \frac{1}{2}[(1-\delta)c_H^{1-\gamma} + \delta c_L^{1-\gamma}]^{\frac{1-\rho}{1-\gamma}}\}^{\frac{\rho-\varepsilon}{1-\rho}}[(1-\delta)c_H^{1-\gamma} + \delta c_L^{1-\gamma}]^{\frac{\gamma-\rho}{1-\gamma}} \end{aligned}$$

As $c_H > c_L > 0$, we have that $Q(\delta) > 0$ for every $\delta \in [0, 1]$.

Moreover, as one can verify by taking the derivative of $S(\delta)$ with respect to δ, the additional restriction $\gamma < \min\{\varepsilon, \rho\}$ is sufficient to ensure that $S'(\delta) < 0$ for every $\delta \in [0, 1]$.

Therefore, we have that

1) If there exists a $\delta^* \in [0, 1]$ such that $S(\delta^*) = 0$, $S(\delta)$ is positive (negative) for all δ smaller (larger) than δ^*. As $Q(\delta)$ is always positive, $W'(\delta)$ has the same sign $S(\delta)$. Henceforth, $W'(\delta)$ is larger/equal/smaller than 0 for δ smaller/equal/larger than δ^*, which implies that W is maximized at $\delta = \delta^*$.

2) If $S(\delta) > 0$ for every $0 \leq \delta \leq 1$, $W'(\delta)$ is always positive, and W is maximized at $\delta = 1$.

3) If $S(\delta) < 0$ for every $0 \leq \delta \leq 1, W'(\delta)$ is always negative, and W is maximized at $\delta = 0$.

By making the appropriate substitutions above, we have:

A) when $\varepsilon = \rho$, $S(1/2) = 0$, and therefore W is maximized at $\delta^* = 1/2$

B) when $\varepsilon > \rho$, $S(1/2) > 0$, which implies either $S(\delta^*) = 0$ at a $\delta^* > 1/2$, or $S(\delta) > 0$ for every $0 \leq \delta \leq 1$. In either case, W is maximized at a δ larger than 1/2.

C) when $\varepsilon < \rho$, $S(1/2) < 0$, which implies either $S(\delta^*) = 0$ at a $\delta^* < 1/2$, or $S(\delta) < 0$ for every $0 \leq \delta \leq 1$. In either case, W is maximized at a δ smaller than $1/2$.

QED.

A.3. A Generalization of the Social Welfare Function

Our isoelastic social welfare function can be generalized to a larger family of social welfare functions. In general terms, social welfare can be written as

$$\widehat{W} = Z\{E_0 J[c_1, E_1 H(c_2)]\} \tag{29}$$

where $Z\{.\}$, $J[.,.]$ and $H(.)$ are continuous and derivable functions. The isoelastic social welfare function in equation (11) is a special case of the above equation, when $Z\{x\} = x^{1-\epsilon}$, $J(x,y) = [x^{1-\rho} + y^{\frac{1-\rho}{1-\gamma}}]^{\frac{1}{1-\rho}}$ and $H(x) = x^{1-\gamma}$.

This extension of our social welfare function is consistent with a general specification of Kreps- Porteus preferences. Specifically, A general way of representing preferences with Kreps-Porteus foundations is

$$U_t = F_t(c_t, E_t U_{t+1}) \tag{30}$$

where U_t is utility at time t, c_t is consumption at time t, E_t is the mathematical-expectation operator conditional on information available at time t, and $F_t(.,.)$ aggregates current consumption and future utility. If the aggregator function $F_t(.,.)$ is linear in its second argument, these preferences are identical to VNM preferences, and the consumer is indifferent to the timing of the resolution of uncertainty. The above equation (29) is consistent with the general specification in equation (30), with $\widehat{W} = U_0$ (where 0 is time behind a veil of ignorance), $Z\{.\} = F_0(E_0 U_1)$, $F_1(c_1,.) = J[c_1,.]$, and $E_1 U_2 = E_1 H(c_2)$.

The results we have obtained for the isoelastic case can be generalize as follows:

1) The generalized Social Welfare Function implies a preference for reversals if and only if

$$Z' \frac{\partial J}{\partial c_1 \partial E_1 H(c_2)} < 0 \tag{31}$$

The above condition is the extension of the Atkinson-Bourguignon condition we introduced in Section 2.

2) By definition, the Social Welfare Function implies a preference for time independence if and only if

$$Z\{E_0J(c_1, E_1H(c_2))\} > Z\{E_0J[c_1, H(c_2)]\} \tag{32}$$

which, by Jensen's inequality, is satisfied for all possible distributions as long as $J(x) \equiv J(c_1, x)$ is concave in x when $Z\prime > 0$, and convex in x when $Z\prime < 0$.

It is immediate to verify that, when a) the condition under 2) is satisfied, b) $Z' \frac{\partial J}{\partial c_1 \partial E_1 H(c_2)} = 0$ and c) $Z\prime \neq 0$, social welfare is maximized with complete time independence (defined as $\delta = 1/2$) in our discrete 2x2 example.

A.4 Quintile Transition Matrices

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Table 1
Quintile Transitions in Germany and U.S.

	1984-1989		1984-1993	
Change in Quintile	**U.S.**	**Germany**	**U.S.**	**Germany**
Down 4	0.5	0.4	0.7	0.6
Down 3	1.8	2.2	2.7	2.9
Down 2	5.5	6	6.6	8.1
Down 1	17.3	18.5	18.3	18
No Change	48.3	44.5	42	36.8
Up 1	19.1	18.9	19.2	20.9
Up 2	6.2	7.1	7.2	8.3
Up 3	1.1	2	2.7	3.3
Up 4	0.1	0.5	0.6	1
	100.0	100.0	100.0	100.0
n	4067	4041	3343	2962

Table 2

Quintile Transition Matricies

		US				
		1993 Quintile				
		1	**2**	**3**	**4**	**5**
	1	0.603	0.223	0.101	0.053	0.021
	2	0.293	0.325	0.210	0.113	0.059
1984 Quintile	**3**	0.119	0.235	0.314	0.225	0.108
	4	0.061	0.150	0.234	0.300	0.255
	5	0.047	0.107	0.123	0.244	0.479

		Germany				
		1993 Quintile				
		1	**2**	**3**	**4**	**5**
	1	0.463	0.293	0.124	0.079	0.042
	2	0.242	0.277	0.268	0.152	0.061
1984 Quintile	**3**	0.160	0.222	0.309	0.205	0.104
	4	0.097	0.181	0.231	0.278	0.214
	5	0.040	0.071	0.106	0.273	0.510

Table 3

Alternative Measure of Mobility

	U.S.	Germany
Mean Absolute Change in Income[1]	0.734	0.477
Reduction in Inequality[2]		
Atkinson (ε=2)		
Average of single year inequality	0.376	0.206
Multiple year inequality	0.302	0.148
Difference	0.074	0.058
Atkinson (ε=4)		
Average of single year inequality	0.669	0.409
Multiple year inequality	0.565	0.284
Difference	0.104	0.125
Correlation	0.591	0.506
Standard Deviation of Residuals around		
Linear predictor	0.541	0.407
Kernel smoothed predictor	0.541	0.401

[1] $\sum|Y_2 - Y_1| / \sum Y_1$ see Fields and Ok (1996).

[2] See Shorrocks (1978b)

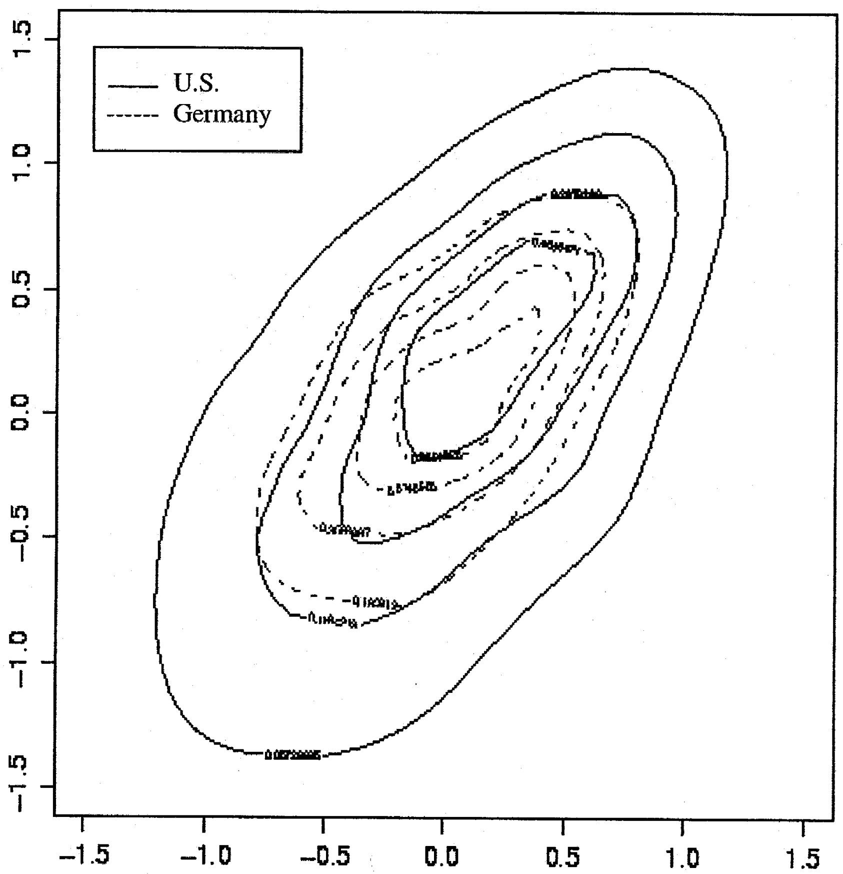
Table 4
Impact of Reversal and Time Independence
$\rho=\gamma=0$

ε	2	4	6
ρ	0	0	0
γ	0	0	0
US			
A_s	0.372	0.666	0.788
A_r	0.302	0.565	0.705
A_o	0.191	0.354	0.459
Reversal (A_r-A_s)	-0.071	-0.101	-0.083
Time independence (A_o-A_r)	-0.110	-0.211	-0.246
Total (A_o-A_s)	-0.181	-0.312	-0.329
Germany			
A_s	0.203	0.401	0.542
A_r	0.148	0.284	0.403
A_o	0.093	0.169	0.229
Reversal (A_r-A_s)	-0.055	-0.117	-0.139
Time independence (A_o-A_r)	-0.055	-0.115	-0.174
Total (A_o-A_s)	-0.110	-0.232	-0.313

Table 5
Impact of Reversal and Time Independence

ε	4	4	4
ρ	2	2	4
γ	0	2	4
US			
A_s	0.668	0.668	0.670
A_r	0.622	0.622	0.670
A_o	0.509	0.578	0.670
Reversal (A_r-A_s)	-0.046	-0.046	0.000
Time independence (A_o-A_r)	-0.114	-0.044	0.000
Total (A_o-A_s)	-0.160	-0.090	0.000
Germany			
A_s	0.406	0.406	0.410
A_r	0.351	0.351	0.410
A_o	0.250	0.310	0.410
Reversal (A_r-A_s)	-0.055	-0.055	0.000
Time independence (A_o-A_r)	-0.101	-0.041	0.000
Total (A_o-A_s)	-0.156	-0.096	0.000

Figure 1
Kernal Smoothed Joint Density of 1984 and 1993 ln Income to Needs Ratio for Germany and the U.S.



[1] Deviations from 1984 and 1993 Means.